**MATH 300. INTRODUCTION TO
MATHEMATICAL REASONING.
FALL 2015.
WEEK 8 (LECTURE 14-15).
PREDICATS AND QUANTIFIERS .**

1. Reading: Sections 1.3,1.6 and Lecture Notes.
2. Home assignment (Due Monday, October 26) (to submit).
  Sect.1.6: 2(a,c,d)4(h,i),6(f,l). Sect.1.7:2f,5a,7b,8b.
  Prove Tautologies

$$(\exists x)(\forall y)P(x,y) \Rightarrow (\forall y)(\exists x)P(x,y);$$

$$(\forall x)(P(x) \wedge Q(x)) \Leftrightarrow (\forall y)P(y) \wedge (\forall z)Q(z);$$

$$(\forall x)(P(x) \vee Q(x)) \Leftarrow (\forall y)P(y) \vee (\forall z)Q(z);$$

$$(\exists x)(P(x) \vee Q(x)) \Leftrightarrow (\exists y)P(y) \vee (\exists z)Q(z);$$

$$(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists y)P(y) \wedge (\exists z)Q(z);$$

but the following formulas are not tautologies (construct counterexamples):

$$(\exists x)(\forall y)P(x,y) \Leftarrow (\forall y)(\exists x)P(x,y);$$

$$(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall y)P(y) \vee (\forall z)Q(z);$$

$$(\exists x)(P(x) \wedge Q(x)) \Leftarrow (\exists y)P(y) \wedge (\exists z)Q(z).$$

  Extra problem. There was a kingdom where lived 30 alchemists. Each of them had one servant and some of servants were dishonest and sold secrets of their owners to all other alchemists. If somebody discovered that his servant was dishonest, he killed this servant next night and everybody learnt about it. It was completely forbidden any communication between alchemists.
  Once the queen issued the order to finish with dishonest servants. We know that there were 7 dishonest servants. On which night after the order will kill the last of dishonest servants ?