Workshop – Math 311 – February 17

1. (Project 1.2 - Chapter 1) Let $a > 0$ and $x_0 > 0$ and define the sequence $\{x_n\}_{n=1}^{\infty}$ recursively by
$$x_n = \frac{1}{2}\left(x_{n-1} + \frac{a}{x_{n-1}}\right).$$
Prove that this sequence converges and find its limit.

2. (a) Let $\{s_n\}$ be a sequence of real numbers and let
$$t_n = \frac{s_1 + s_2 + \cdots + s_n}{n}, \qquad n \geq 1.$$
Prove that if $\lim_{n\to\infty} s_n = s$ then $\lim_{n\to\infty} t_n = s$. Give an example to show that $\{t_n\}$ may converge even though $\{s_n\}$ does not.

   (b) Use part (a) to show that if $x_{n+1} - x_n \to x$ then $\dfrac{x_n}{n} \to x$ for a sequence of real numbers $\{x_n\}$.

3. (alternatively you could consider Project 1.3 - Chapter 1) Let a be a positive real number. Define a sequence $\{x_n\}$ by
$$x_0 = 0, \quad x_{n+1} = a + x_n^2, \qquad n \geq 0.$$
Find a necessary and sufficient condition on a in order that a finite $\lim_{n\to\infty} x_n$ should exist.

4. Prove that $[0,1]$ is uncountable and hence so is $\mathbb{R}$. (You could follow Project 1.4 - Chapter 1 or use a different proof).