What you should learn from Recitation 3: First order linear ODE

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Disclaimer

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- These slides may suffer from errors. Please use them with your own discretion since debugging is beyond the author's ability.

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- Prove that
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and use direct substitution to get
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$$y' + \frac{2t-1}{t(t^2-4)}y = \frac{3t-5}{t(2t+1)}.$$

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is guaranteed to have a unique solution.

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State where in the ty-plane the hypothesises of Theorem 2.4.2 are satisfied for

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- When $f(t, y) = (t^2 + y^2)^{3/2}$, one can see that $f_y(t, y) = \frac{3}{2}(t^2 + y^2)^{3/2}$. Both f and f_y are continuous on all the points on ty-plane. So Theorem 2.4.2 are satisfied everywhere.

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- Therefore the points where Theorem 2.4.2 works can be described by the set

$$\{(t, y) : y + t - 1 > 0\}$$

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- If $0 < y < 2$, then $y' = y^2(y-2)(y-3) > 0$. So $y = 2$ is stable from below. Therefore $y = 2$ is stable.
- Same reason as above, $y = 0$ is unstable from above.
- If $y < 0$, then $y' = y^2(y-2)(y-3) > 0$. So $y = 0$ is stable from below. Therefore $y = 0$ is semistable.

Quiz Problem 2

Find out the equilibrium solutions of the ODE

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Answer: There are three equilibrium solutions $y = 0, y = 2$ and $y = 3$.

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Challenging Problem last week

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$$y' = \sin y$$

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- If $2k\pi < y < 2k\pi + \pi$, we have $\sin y > 0$.

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- If $2k\pi + \pi < y < 2k\pi + 2\pi$,

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- If $2k\pi + \pi < y < 2k\pi + 2\pi$, we have $\sin y < 0$. So $y = 2k\pi + \pi$ is stable from above; $y = 2k\pi + 2\pi$ is unstable from below.
- Therefore the equilibrium solution $y = 2k\pi + \pi$ is stable

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- If $2k\pi + \pi < y < 2k\pi + 2\pi$, we have $\sin y < 0$. So $y = 2k\pi + \pi$ is stable from above; $y = 2k\pi + 2\pi$ is unstable from below.
- Therefore the equilibrium solution $y = 2k\pi + \pi$ is stable (I believe you don't have doubts)

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- If $2k\pi + \pi < y < 2k\pi + 2\pi$, we have $\sin y < 0$. So $y = 2k\pi + \pi$ is stable from above; $y = 2k\pi + 2\pi$ is unstable from below.
- Therefore the equilibrium solution $y = 2k\pi + \pi$ is stable (I believe you don't have doubts) and the equilibrium solution $y = 2k\pi$ is unstable

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- If $2k\pi + \pi < y < 2k\pi + 2\pi$, we have $\sin y < 0$. So $y = 2k\pi + \pi$ is stable from above; $y = 2k\pi + 2\pi$ is unstable from below.
- Therefore the equilibrium solution $y = 2k\pi + \pi$ is stable (I believe you don't have doubts) and the equilibrium solution $y = 2k\pi$ is unstable (you have to think a little bit).

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Exact Equations: Brief Recall

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or in simpler notation:
$$M_y(x,y)=N_x(x,y).$$
- In this case, there exists a function $F(x,y)$, such that
$$F_x(x,y)=M(x,y), F_y(x,y)=N(x,y)$$
and
$$F(x,y(x))=C$$

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gives the general implicit solution of the ODE.

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- In this case, there exists a function $F(x,y)$, such that
$$F_x(x,y) = M(x,y), F_y(x,y) = N(x,y)$$
and
$$F(x, y(x)) = C$$
gives the general implicit solution of the ODE.
- So in order to solve an exact ODE,

Exact Equations: Brief Recall

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is called exact if
$$\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x},$$
or in simpler notation:
$$M_y(x,y) = N_x(x,y).$$
- In this case, there exists a function $F(x,y)$, such that
$$F_x(x,y) = M(x,y), F_y(x,y) = N(x,y)$$
and
$$F(x, y(x)) = C$$
gives the general implicit solution of the ODE.
- So in order to solve an exact ODE, it suffices to find such a function.

Exact Equations: Brief Recall

How to find $F(x, y)$?

Exact Equations: Brief Recall

How to find $F(x, y)$?

1. Make sure your equation is exact first!

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How to find $F(x, y)$?

1. Make sure your equation is exact first! If it were not exact, don't attempt the following steps!
2. Assuming the equation $M(x, y) + N(x, y)y' = 0$ is exact.

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1. Make sure your equation is exact first! If it were not exact, don't attempt the following steps!
2. Assuming the equation $M(x, y) + N(x, y)y' = 0$ is exact. Since

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2. Assuming the equation $M(x, y) + N(x, y)y' = 0$ is exact. Since

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we integrate both sides with respect to x,

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$$F_x(x, y) = M(x, y),$$
we integrate both sides with respect to x, to get
$$F(x, y) = \int M(x, y)dx + SomethingThatDoesNotDependOnx$$

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But $SomethingThatDoesNotDependOnx$ means a function on y.

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But $SomethingThatDoesNotDependOnx$ means a function on y. So we call it $\phi(y)$.

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But $SomethingThatDoesNotDependOnx$ means a function on y. So we call it $\phi(y)$.
3. Now use the fact that
$$F_y(x, y) = N(x, y),$$

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3. Now use the fact that
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and this leaves us,

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How to find $F(x, y)$?

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But $SomethingThatDoesNotDependOnx$ means a function on y. So we call it $\phi(y)$.

3. Now use the fact that

$$F_y(x, y) = N(x, y),$$

and this leaves us, after the algebra,

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How to find $F(x, y)$?

1. Make sure your equation is exact first! If it were not exact, don't attempt the following steps!
2. Assuming the equation $M(x, y) + N(x, y)y' = 0$ is exact. Since
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we integrate both sides with respect to x, to get
$$F(x, y) = \int M(x, y)dx + SomethingThatDoesNotDependOnx$$
But $SomethingThatDoesNotDependOnx$ means a function on y. So we call it $\phi(y)$.
3. Now use the fact that
$$F_y(x, y) = N(x, y),$$
and this leaves us, after the algebra, with some expression concerning $\phi'(y)$.

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1. Make sure your equation is exact first! If it were not exact, don't attempt the following steps!
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we integrate both sides with respect to x, to get
$$F(x, y) = \int M(x, y)dx + SomethingThatDoesNotDependOnx$$
But $SomethingThatDoesNotDependOnx$ means a function on y. So we call it $\phi(y)$.
3. Now use the fact that
$$F_y(x, y) = N(x, y),$$
and this leaves us, after the algebra, with some expression concerning $\phi'(y)$. Then you may get $\phi(y)$ by integration.

Exact Equations: Brief Recall

What if your equation is not exact?

Exact Equations: Brief Recall

What if your equation is not exact?

- In some cases,

Exact Equations: Brief Recall

What if your equation is not exact?

- In some cases, you can find an integating factor

Exact Equations: Brief Recall

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- When

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is independent of y and depends ONLY ON x, then you can find an integrating factor by solving the differential equation

$$\frac{M_y - N_x}{N} = \frac{\mu'(x)}{\mu(x)}$$

Example: Book Problem 2.6.20

Use the given integrating factor

$$\mu(x, y) = ye^x$$

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Use the given integrating factor

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to solve the nonexact ODE

$$\left(\frac{\sin y}{y} - 2e^{-x}\sin x\right) + \left(\frac{\cos y + 2e^{-x}\cos x}{y}\right) y' = 0.$$

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So $M_y = N_x$

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- Check that you have an exact equation.

$$\begin{aligned} M &= e^x \sin y - 2y\sin x \\ M_y &= e^x\cos y - 2\sin x \\ N &= e^x\cos y + 2\cos x \\ N_x &= e^x\cos y - 2\sin x. \end{aligned}$$

So $M_y = N_x$ and yes you do get an exact ODE.

Example: Book Problem 2.6.20

- Integrate M with respect to x:

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\begin{aligned}
F(x,y) &= \int M(x,y)dx = \int (e^x \sin y - 2y \sin x)dx \\
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- So the final (implicit) solution of our ODE is

$$e^x \sin y + 2y \cos x = C.$$

Example: Book Problem 2.6.25

Find an integrating factor of the ODE

$$(3x^2y + 2xy + y^3) + (x^2 + y^2)y' = 0.$$

and solve it.

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  Obvious enough that $M_y \neq N_x$.

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$$\frac{M_y - N_x}{N}$$

Example: Book Problem 2.6.25

- Compute $(M_y - N_x)/N$:

$$\frac{M_y - N_x}{N} = \frac{3x^2 + 2x + 3y^2 - 2x}{x^2 + y^2}$$

Example: Book Problem 2.6.25

- Compute $(M_y - N_x)/N$:

$$\frac{M_y - N_x}{N} = \frac{3x^2 + 2x + 3y^2 - 2x}{x^2 + y^2} = 3.$$

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$$\frac{\mu'(x)}{\mu(x)} = 3$$

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$$\begin{aligned} \frac{\mu'(x)}{\mu(x)} &= 3 \\ \ln \mu(x) &= \int 3dx = 3x \end{aligned}$$

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- Get your exact equation:

$$e^{3x}(3x^2y + 2xy + y^3) + e^{3x}(x^2 + y^2)y' = 0.$$

Example: Book Problem 2.6.25

- Integrate your new M(x,y):

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- Integrate your new M(x,y):

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Example: Book Problem 2.6.25

- Integrate your new M(x,y):

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\begin{aligned}
F &= \int e^{3x}(3x^2y+2xy+y^3)dx = \frac{1}{3}\int (3x^2y+2xy+y^3)de^{3x} \\
&= \frac{1}{3}(3x^2y+2xy+y^3)e^{3x} - \frac{1}{3}\int e^{3x}(6xy+2y)dx
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&= \frac{1}{9}(9x^2y + 6xy + 3y^3 - 6xy - 2y)e^{3x} + \frac{2}{3}y\int e^{3x}\cdot dx
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&= \frac{1}{9}(9x^2y+3y^3-2y)e^{3x} + \frac{2}{9}ye^{3x} + \phi(y)
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&= \frac{1}{3}(3x^2y + y^3)e^{3x} + \phi(y)
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Example: Book Problem 2.6.25

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- Use $F_y(x,y) = N(x,y)$ to get $\phi(y)$:

$$F_y(x,y) = \frac{1}{3}(3x^2 + 3y^2) + \phi'(y) = N(x,y) = x^2 + y^2$$
$$\Rightarrow \quad \phi'(y) = 0$$

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- Use $F_y(x, y) = N(x, y)$ to get $\phi(y)$:

$$\begin{aligned} F_y(x,y) &= \frac{1}{3}(3x^2 + 3y^2) + \phi'(y) = N(x,y) = x^2 + y^2 \\ \Rightarrow \quad & \phi'(y) = 0 \end{aligned}$$

So just take $\phi(y) = 0$.

Example: Book Problem 2.6.25

- Use $F_y(x, y) = N(x, y)$ to get $\phi(y)$:

$$F_y(x, y) = \frac{1}{3}(3x^2 + 3y^2) + \phi'(y) = N(x, y) = x^2 + y^2$$
$$\Rightarrow \quad \phi'(y) = 0$$

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Graded Homework Problem: 2.6.8.

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The End