REVIEW QUESTIONS FOR MIDTERM 2

Note: You don't have to answer these question verbally. Just make sure you know the answers in your mind and make sure you are capable on solving example problems from the book.

1. HIGHER ORDER LINEAR ODE

(1) When does the IVP

$$y^{(n)} + p_1(t)y^{(n-1)} + \cdots + p_n(t)y = g(t); y(0) = y_0, \cdots, y^{(n-1)}(0) = y_0^{(n-1)}$$

has a unique solution? How to find the maximal interval of existence? (Theorem 4.1.1 Book Problem 4.3.1 to 4.3.6)

(2) What is a fundamental set of solution? What does it mean for functions $y_1(t), \cdots, y_n(t)$ to be linearly dependent or linearly independent? (Theorem 4.1.2, Example 1 and 2 in Section 4.1, Theorem 4.1.3, Book Problem 4.1.7 to 4.1.10)

(3) What is a Wronskian and how is it related to linear dependency and independency? How to compute the Wronskian for a given set of functions? (Theorem 4.1.2, Book Problem 4.1.7 to 4.1.17)

(4) What is the structure of solutions for nonhomogeneous linear ODE? What is the complementary solution and what is a particular solution? (Read the last subsection of 4.1)

(5) For a homogeneous linear ODE with constant coefficients, what is its characteristic equation? How does its root determine the general solution? What happens when you have repeated root and how to deal with complex roots? How to discuss the long-term behavior? (Section 3.1, 3.3, 3.4, 4.2, Recitation Note 7-1, Recitation Note 9)

(6) What is Euler's equation? How to solve them? (Section 5.4, Recitation Note 7-1)

(7) To find a particular solution for the nonhomogeneous equation

$$a_n y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = g(t)$$

what is the first try template if
  - $g(t)$ is a constant function? e.g. $g(t) = 3$?
  - $g(t)$ is a polynomial function? e.g. $g(t) = 2t^2 + 3t + 3$ or $g(t) = 5t^2$ (Be careful not to miss terms)
  - $g(t)$ is a trigonometric function? e.g.$g(t) = \cos 2t$ or $g(t) = \sin t$ (Be careful not to miss terms)
  - $g(t)$ is an exponential function? e.g. $g(t) = e^{3t}$ or $g(t) = e^{3t}$.
  - $g(t)$ is a product of exponential functions and polynomial functions? e.g. $g(t) = t^2 e^t$ (Be careful not to miss terms)

- $g(t)$ is a product of trigonometric functions and polynomial functions? e.g. $g(t) = t^2 \sin t$ (Be very careful since there should be a lot of terms)
- $g(t)$ is a product of exp, trig and polynomial? e.g. $g(t) = te^t \cos t$
- $g(t)$ is a sum of the above types? e.g. $g(t) = 3 + 2e^{-t} + \sin 2t$. (Table 3.5.1, Recitation Note 8)

(8) When is your first try template fail? And how many tries will fail? How should you modify your template? How to solve the coefficients? (Section 3.5, 4.3, Recitation Note 8)

(9) What does variation of perimeter do? How to perform the procedure to second order nonhomogeneous linear ODE, using either integral formula or linear equations? (Section 3.6, Recitation Note 8)

(10) How to describe vibrations? What is the the equation of motion in general? (Section 3.7, Formula (7), (8))

(11) What is free vibration? What are damped and undamped vibrations? What does it mean for a vibration to be underdamped, critically damped and overdamped? (Section 3.7, Formula (11), (21), (23), (24), (25))

(12) For the free undamped vibration, what are natural frequency, amplitude and phase? For the free underdamped vibration, what is the quasi-frequency and quasi-period? How to obtain these features in each case? (Section 3.7, Formula (16), (17))

(13) What is forced vibration? For the forced damped vibration, what are the steady state solution and the transient solution? How to identify them and graph them? For the forced undamped vibration, when does resonance or beat occur? (Section 3.8, Formula (9), Figure 3.8.1, Example 3, Example 4)

2. Linear algebra

(1) How to covert a higher order differential equation into a first order linear system? (Section 7.1, Example 1)

(2) How to add and multiply two matrices? How to multiply a number to a matrix? What is the transpose, conjugate or adjoint of a matrix? How to multiply a matrix to a vector? What is an identity matrix? How to compute the determinant of a matrix? What is the inverse to a matrix and how to compute it? (Section 7.2)

(3) How to solve a system of linear equations? How to use the augmented matrix to proceed? What happens if the matrix of the (homogeneous or nonhomogeneous) system is nonsingular? What happens if the matrix of the (homogeneous or nonhomogeneous) system is singular? How does the solution look like? (Section 7.3, Example 1, 2)

(4) How to determine if a set of vectors is linear dependent or independent? If linear dependent, how to find a relation? (Section 7.3, Example 3)

(5) How to find the eigenvalues and eigenvectors to a given matrix? (Section 7.3, Example 4)

(6) What is the structure of the solutions to a (homogeneous) linear system of ODE? What is a fundamental set of solutions and how to verify if you have a fundamental set of solutions? (Section 7.4, Theorem 7.4.1 to 7.4.4)