

QUIZ 4 FOR CALC4 ON SEPT. 25, 2014

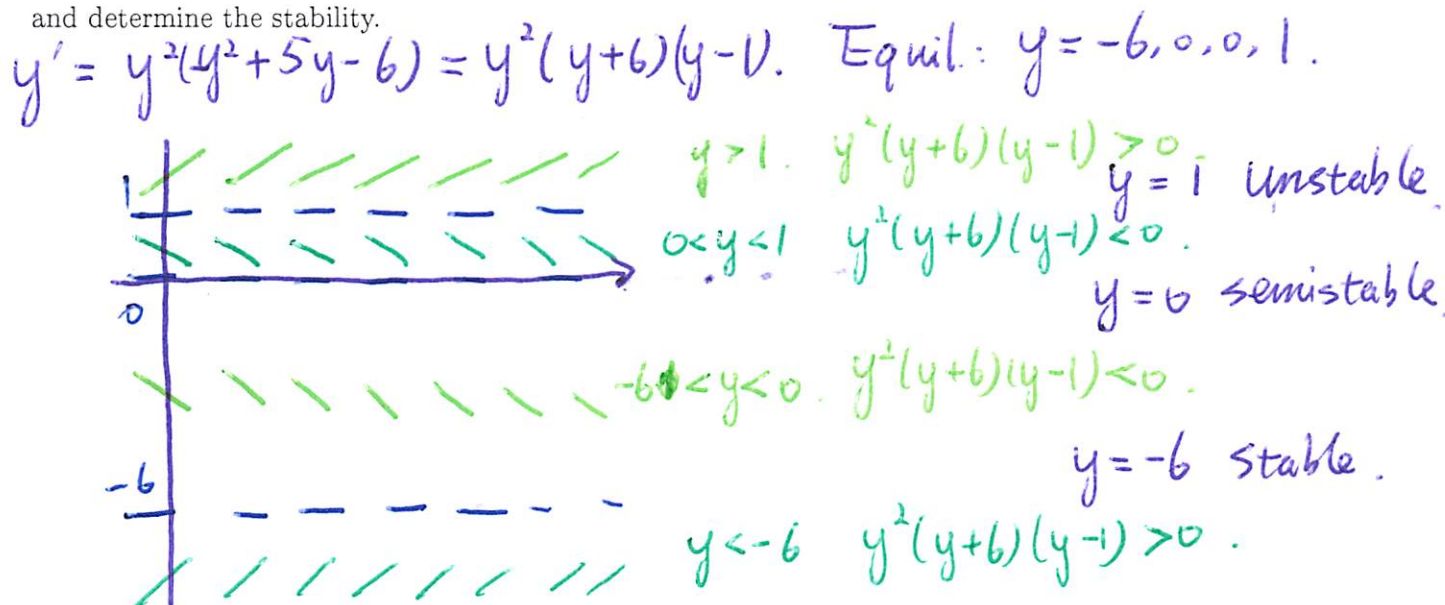
Name: Solution RUID: _____

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(1) Find the equilibrium of the autonomous ODE

$$y' = y^2(y^2 + 5y - 6)$$

and determine the stability.



(2) Use the given integrating factor to solve the following ODE

$$3xy + y^2 + (x^2 + xy)y' = 0, \mu(x) = x$$

Check your answers!

Multiply $\mu(x)$ to the ODE $\Rightarrow \underbrace{3x^2y + xy^2}_M + \underbrace{(x^3 + x^2y)}_N y' = 0$.

$M_y = 3x^2 + 2xy$, $N_x = 3x^2 + 2xy$. $M_y = N_x$ exact!

$\psi(x, y) = \int (3x^2y + xy^2) dx = x^3y + \frac{1}{2}x^2y^2 + \varphi(y)$

$\frac{\partial \psi}{\partial y} = x^3 + x^2y + \varphi'(y) = N = x^3 + x^2y \Rightarrow \varphi'(y) = 0$

$\Rightarrow \varphi(y) = 0 \Rightarrow \psi(x, y) = \boxed{x^3y + \frac{1}{2}x^2y^2 = C}$

CHECK: $\frac{d}{dx} (x^3y + \frac{1}{2}x^2y^2) = 3x^2y + x^3 \cdot y' + xy^2 + x^2 \cdot y \cdot y' = 0$
 $3x^2y + xy^2 + (x^3 + x^2y)y' = 0 \cdot \checkmark$