QUIZ 3 FOR CALC 4 ON SEPT. 18, 2014

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(1) Find the solution of the following initial value problem IN EXPLICIT FORM

$$y^2(1-x^2)^{1/2}dy = \arcsin x dx, y(0) = 1$$

$y^2 dy = \frac{\arcsin x}{\sqrt{1-x^2}} dx$.

Integrate:

$\frac{1}{3}y^3 = \int \arcsin x \, d(\arcsin x)$

$= \frac{1}{2}(\arcsin x)^2 + C$.

$y = \sqrt[3]{\frac{3}{2}(\arcsin x)^2 + C}$.

$y(0) = 1 \Rightarrow C = 1$.

Solution:

$y(x) = \sqrt[3]{\frac{3}{2}(\arcsin x)^2 + 1}$

(2) Can you conclude if the following initial value problem

$$y'(t) = \sqrt{y+t-1}, y(0) = 1$$

has a unique solution on *some* interval around the initial time $t = 0$?

$f(t,y)$ (= $\sqrt{y+t-1}$) in fact is not continuous NEAR $(0,1)$. This requires advanced knowledge.

Nevertheless, $\frac{\partial f}{\partial y}(t,y) = \frac{1}{2\sqrt{y+t-1}}$ blows up at $(0,1)$. Therefore ~~the~~ we cannot conclude the existence of a unique solution.