Math 244 Exam 2 Practice Problems

1. Consider the initial value problem

$$4y'' + 12y' + 9y = 0, \;\; y(0) = 1, y'(0) = -4$$

   (a). Find the solution.

   (b). Change the second initial condition to $y'(0) = \beta$ and find the solution as a function of β. Then find the critical value of β that separates solutions that always remain positive from those that eventually become negative.

2.

a) Find the general solution of the differential equation
$y^{(4)} - y''' - y'' + y' = 0$

b) Determine a suitable form of the particular solution of the $y^{(4)} - y''' - y'' + y' = t^2 + 4 + t\sin t$. (You do not have to find the constants).

3. Use the variation of parameters method to find the general solution of: $y'' - 2y' + y = \frac{e^t}{t^2+1}$

4. A mass weighing 4 lb stretches a spring 1.5 inches. The mass is displaced 2 inches in the positive direction from the equilibrium position and released. There is no damping and the mass is acted upon by an external force $F_0 = 2\cos 3t$ lbs.

   a) Find the equation of motion.

   b) If the external force is replaced by a force $F_0 = 4\sin\omega t$ find the value of ω for which resonance occurs.

5. A 3.2 pound weight is attached to a spring with stiffness (i.e. spring constant) $k = 2$, and the system is then immersed in a medium that imparts a damping force equal to 0.4 times the velocity. *(Assume g= 32 ft per sec^2)*

a) Find the equation of motion if the weight is released from rest 1 foot above the equilibrium.

b) Find the (quasi) frequency and the period.

c) Draw a rough sketch of the solution.

6.

a) Show that the following vectors are linear dependent by obtaining a linear relationship between them.

$$\vec{x}^{(1)} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \quad \vec{x}^{(2)} = \begin{pmatrix} -4 \\ 1 \\ -6 \end{pmatrix}, \quad \vec{x}^{(3)} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

b) Show that the following functions are linear independent by calculating their Wronskian.

$y_1 = t,\ y_2 = sint,\ y_3 = cost$