**MATH 244:** **Name:** ______________________

**Fall 2014**

**Exam 1**

**10/06/2014**

**Time: 60 Minutes** Section __________

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This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator or electronic device on this exam.

You are required to show your answers clearly for each problem on this exam.

Grade Table

| Question | Points | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 10 | |
| 4 | 20 | |
| 5 | 20 | |
| 6 | 10 | |
| Total: | 100 | |

1. (20 points) Consider the initial value problem

$$\frac{dy}{dt} = \frac{1 + y^4}{4(t+2)y^3}, \quad y(0) = 1$$

(a). Solve for $y$. Express your solution in the form $y = F(t)$.

(b). What is the interval of existence?

2. (20 points) Consider the initial value problem:

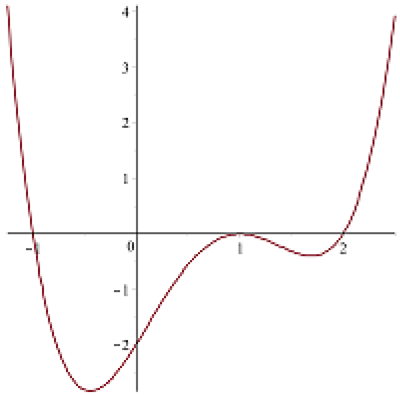
$$3x^2 + 2xy^2 + 2x^2y\frac{dy}{dx} = 0\,, \quad y(2) = -3$$

(a) Show that the equation is exact.

(b) Solve the equation and find an explicit expression for $y$.

(c) Find the interval of existence for the solution.

3. (10 points) Consider the differential equation $\frac{dy}{dx} = f(y)$ where the graph of $f(y)$ given below.




(a). Draw the phase line (use the space above)

(b). Sketch a few solutions showing the behavior near equilibrium points and asymptotic behavior

(c) Determine the equilibrium points and classify them as asymptotically stable, unstable or semi-stable.

4. (20 points) Consider the differential equation

$$y'' - 2y' - 8y = 0\,, \quad y(0) = \alpha, y'(0) = 2$$

(a). Solve the initial value problem.

(b). The solution can exhibit 3 possible long term behaviors depending on the value of $\alpha$. Determine them (along with the corresponding $\alpha$ values).

5. (20 points) According to Newton's law of cooling the rate of change of the temperature $T$ of an object with respect to time $t$ is given by

$$\frac{dT}{dt} = -k(T - T_a)$$

where $T_a$ is the ambient (or room) temperature and $k$ is a positive constant.
A pot of liquid is put on the stove to boil. The temperature of the liquid reaches 170 °F and then the pot is taken off the burner and placed on a counter in the kitchen. The temperature of the air in the kitchen is 76 °F.

(a). Obtain an expression for $T(t)$, the temperature of the liquid at time $t$. *(Your expression will include $k$)*

(b). After two minutes the temperature of the liquid in the pot is 123 °F. Find an expression for $k$

6. (10 points) Suppose you are trying to solve $y' = \sqrt{t+y}$, $y(0) = 1$ numerically.

a) If you use the Euler method what will be the local truncation error? Give your answer in terms of $t$, step size $h$ and the solution $\Phi$.

b) Suppose you tried the Runge-Kutta method with step size $h = 0.1$ and got an error of $2 \times 10^{-1}$ . What do you expect the error to be if you change the step size to 0.01?