4.

a. (10 points) If $x^3 + 3x^2y - 4y^2 = 16$, find $\dfrac{dy}{dx}$ at $(2, 2)$.

$\frac{dy}{dx} =$	

b. (8 points) If $y = x^{\sin x}$, $0 < x < \pi$, find $\dfrac{dy}{dx}$ as a function of x.

$\frac{dy}{dx} =$	

5. (17 points) The length of a rectangle is increasing at 4 ft/min and its width is decreasing at 3 ft/min. At what rate is the area of the rectangle changing when its length is 9 ft and its width is width is 5 ft? Is the area of the rectangle increasing or decreasing?

6. (18 points) Let $f(x) = 2\sqrt{3x+3}$. Use linear approximation or differentials to approximate $f(2.03)$. You do not need to simplify your answer.

9. (18 pts) Find the intervals where the function $y = x - \dfrac{4}{x^2}$ is increasing and decreasing, concave up, and concave down. Find all horizontal, vertical asymptotes, relative extrema, and inflections. Write "none" in the blank if there are none.

Increasing	
Decreasing	
Concave up	
Concave down	
Horizontal asymptotes	
Vertical asymptotes	
Relative maxima	
Relative minima	
Inflections	

10. (9 points each)

a. Find $\lim\limits_{x \to 1} \dfrac{x^{10}-1}{x^3-1}$.

$\lim\limits_{x \to 1} \dfrac{x^{10}-1}{x^3-1} =$	

b. Find $\lim\limits_{x \to 0} \dfrac{\sin(x^2)}{\cos(3x)-1}$.

$\lim\limits_{x \to 0} \dfrac{\sin(x^2)}{\cos(3x)-1} =$	

2. (9 points each) Find the following limits, giving reasons for your answers. You may use any method from this course.

a. $\lim_{x \to \pi} \dfrac{x - \pi}{x^3 - \pi^3} =$ ____________________

b. $\lim_{x \to 0} \dfrac{e^{3x} - 3x - 1}{x^2} =$ ____________________

4.

a. (10 points) If $x^3 + 2xy + y^3 = 13$, find $\dfrac{dy}{dx}$ at $(2, 1)$. ______________________________

b. (8 points) If $y = x^{2x^2}$, find $\dfrac{dy}{dx}$ as a function of x. $\dfrac{dy}{dx} =$__________________________

5. (17 points) At noon, a car traveling north at 45 mi/hr is 20 miles north of a truck traveling east at 35 mi/hr. At what rate will the distance between them be changing 3 hours later? You don't have to multiply out the numbers that occur in this problem.

6. (18 points) Let $f(x) = x^3(x+1)^4$ at $x = 1$. Use linear approximation or differentials to estimate $f(1.02)$.

8. (17 pts)

a. (9 points) Find $\lim_{x\to\infty} \dfrac{e^{2x}+4x}{2e^{2x}+x}$. Answer =________________________

b. (8 points) Find $\lim_{x\to-\infty} \dfrac{e^{2x}+4x}{e^{2x}+x}$. Answer = ________________________

14. (18 points) Consider the function $f(x) = x + (4/x)$. For this function, there are four important intervals, $(-\infty, A]$, $[A, B)$, (B, C), and $[C, \infty)$, where A and C are the critical numbers and $f(x)$ is not defined at B.

Find A:	
Find B:	
Find C:	
The function is increasing on the interval(s):	
The function is decreasing on the interval(s):	
The function is concave up on the interval(s):	
The function is concave down on the interval(s):	
There are inflection(s) at:	
There is a local maximum at:	

4. (18 points) If $x^3 + xy + y^2 = 7$, find $\dfrac{dy}{dx}$ at $(1, 2)$. ______________________________

5. (17 points) Prove that the equation $\dfrac{1}{x+1} = x^2 - x - 1$ has at least one solution on the interval $(1, 2)$.

6a. (9 points) Find $\lim_{x\to 0} \frac{\sin^2 x}{\sin(2x^2)}$.

Answer	

6b. (9 points) Find $\lim_{x\to 1} \frac{\ln(x^2+2) - \ln(3)}{x-1}$.

Answer	

8. (17 pts) A car traveling north at 40 mi/hr and a truck traveling east at 30 mi/hr leave an intersection at the same time. At what rate will the distance between them be changing 4 hours later?

Answer:	

9. (18 points) Find the absolute maximum and minimum of the function

$$f(x) = (6x + 1)e^{3x}$$

on the interval $[-1000, 1000]$. Hint: You can figure this out without a calculator if you use the first derivative test and think about the signs at the endpoints.

maximum:	
minimum:	

10. (18 points) The marginal revenue of a certain commodity is

$$R'(x) = -3x^2 + 4x + 32$$

where x is the level of production in thousands. Assume $R(0) = 0$. Find $R(x)$. What is the demand function $p(x)$? Find the level of production that maximizes revenue. You do not need to check that your answer is a maximum.

R(x):	
p(x):	
x that maximizes revenue:	

12. (18 points) Use linear approximation or differentials to approximate $(8.02)^{1/3}$.

Approx value of $(8.02)^{1/3}$**:**	

13. (6 points apiece) A manufacturer of car batteries estimates that the fraction of his batteries will work for at least t months is

$$p(t) = e^{-0.03t}.$$

a. What fraction of the batteries can be expected to last at least 40 months?
b. What fraction can be expected to fail before 50 months?
c. What fraction can be expected to fail between the 40th and 50th months?

Last 40 months:	
Fail before 50 months:	
Fail between 40 and 50 months:	

14. (18 points) Consider the function $f(x) = \dfrac{(x-1)^2}{(x+2)(x-4)}$. For this function, we have $f'(x) = \dfrac{-18(x-1)}{(x+2)^2(x-4)^2}$ and $f''(x) = \dfrac{54((x-1)^2+3)}{(x+2)^3(x-4)^3}$.

Horizontal asymptotes:	
Find vertical asymptotes:	
The function is increasing on the interval(s):	
The function is decreasing on the interval(s):	
The function is concave up on the interval(s):	
The function is concave down on the interval(s):	
There are inflection(s) at:	
There are local maxima at:	
There are local minima at:	

Sketch the graph below