① Binomial theorem:

$$(a+b)^0 = 1$$

$$(a+b)^1 = a+b.$$

$$(a+b)^2 = a^2+2ab+b^2$$

$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3.$$

$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4.$$

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

Exercise: Give me $(a+b)^5$.

Answer: $(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

Extra Ex. $(a+b)^6 = ?$

So you know how to deal with $(a+b)^n$.

② Fractions

Ex.: $\frac{3}{5} + \frac{1}{6} = ?$

$$= \frac{18+5}{30} = \frac{23}{30}.$$

Ex: $\frac{a}{b} + \frac{c}{d} = ?$

Find the common denominator:

$$\cancel{\frac{\quad}{bd}} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}.$$

Exercise: $\frac{x}{x+1} + \frac{x+1}{x} = ?$

$$= \frac{x \cdot x}{(x+1)x} + \frac{(x+1)(x+1)}{x(x+1)} = \frac{x^2 + (x+1)^2}{(x+1)x}.$$

$$= \frac{x^2 + x^2 + 2x + 1}{x^2 + x} = \frac{2x^2 + 2x + 1}{x^2 + x}. \#$$

$$\left(= \underbrace{\frac{2x^2+2x}{x^2+x}}_{\shortparallel\ \frac{2\cancel{(x^2+x)}}{\cancel{x^2+x}}} + \frac{1}{x^2+x} = 2 + \frac{1}{x^2+x} \right).$$

③ Quadratic formula:

Webwork ex. Solve $x^2 - 8x + 15 = 0$.

Recall we have the quadratic formula

$$\cancel{x = \frac{\quad}{2}}$$

For $ax^2 + bx + c = 0$,

$$\Delta = b^2 - 4ac.$$

Roots: $x_1 = \dfrac{-b+\sqrt{\Delta}}{2a}$, $x_2 = \dfrac{-b-\sqrt{\Delta}}{2a}$

$$\Delta = 8^2 - 4 \times 1 \times 15 = 64 - 60 = 4 \Rightarrow \sqrt{\Delta} = 2.$$

$$x_1 = \frac{-(-8)+2}{2 \times 1} = \frac{10}{2} = 5.$$

$$x_2 = \frac{-(-8)-2}{2 \times 1} = \frac{6}{2} = 3.$$

Rmk: ① There are other ways playing with such eqn. But quad. formula works ALWAYS.

② The way works always is not always the easiest way. The easiest way is to factorize.

Remark: ③ If x_1 and x_2 are the roots to the quad. eqn. $ax^2+bx+c=0$.

Then $ax^2+bx+c=a(x-x_1)(x-x_2)$

Ex. $x^2-x-1=0$.

Using quad. formula,

$$x_1=\frac{1+\sqrt{1^2+4\times 1}}{2}=\frac{1+\sqrt{5}}{2}. \quad x_2=\frac{1-\sqrt{5}}{2}$$

$$\Rightarrow x^2-x-1=\left(x-\frac{1+\sqrt{5}}{2}\right)\left(x-\frac{1-\sqrt{5}}{2}\right).$$

§1.1. postponed to Chap. 2.

§1.2.

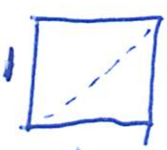
① Real numbers:

natural numbers: 0, 1, 2, 3, ...

integers: ..., -2, -1, 0, 1, 2, 3, ...

rational number: $\frac{m}{n}$, m, n integers.

$\sqrt{2} \neq \frac{m}{n}$.

$\sqrt{2}$ is the length of the diagonal of 

irrational number: "completion" of rational #s.

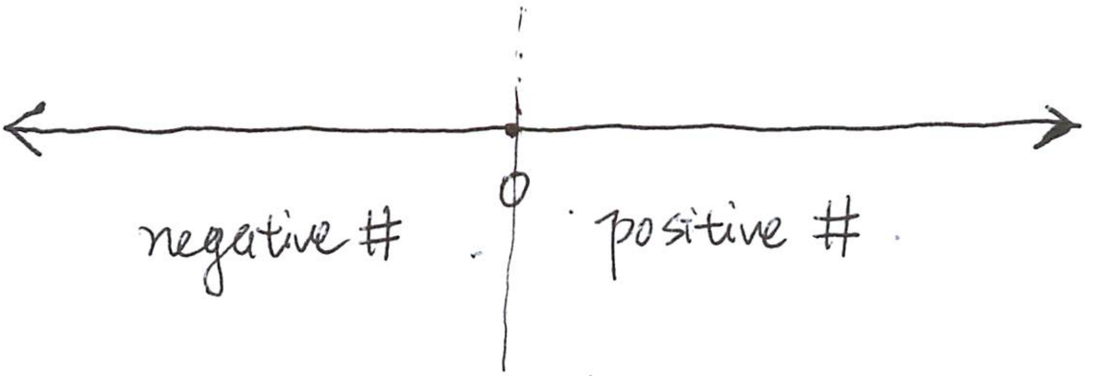
written in decimals, it's infinite & non repeating

Real # is the collection of rat'l # & irrat'l #.

Real # is in one-to-one correspondence to geometric length.

Rigorous def'n in $18^{th} \sim 19^{th}$ century.

In other words, a real # can represented as a point on ~~the~~ a line, called real number line.



② Distance of two real #s.

Let a, b be two real #s, then one of the following happens:

$a < b$, $a = b$, $a > b$.

When $a < b$, distance between a & b = $b - a$.

$a = b$ " " " $= 0$.

$a > b$ " " " $= a - b$.

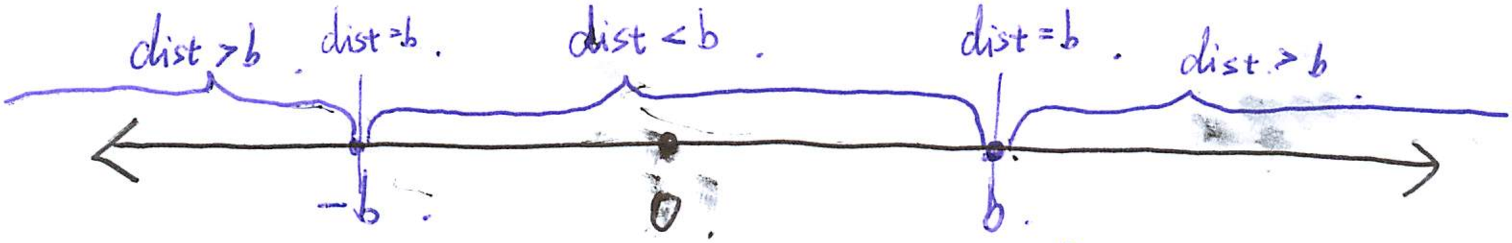
③ Use absolute value to ~~g~~ describe distance.

~~For~~ a real # x, the abs. value is defined as

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

So $|x| \geq 0$.

Geometrically, this gives the characterization of the distance between a and 0.



Prop. 7 ⇒ dist. of a & $0 = b$ ~~iff~~ ⇔ $a = \pm b$.

Prop. 8 ⇒ 〃 〃 〃 $< b$ ⇔ $-b < a < b$

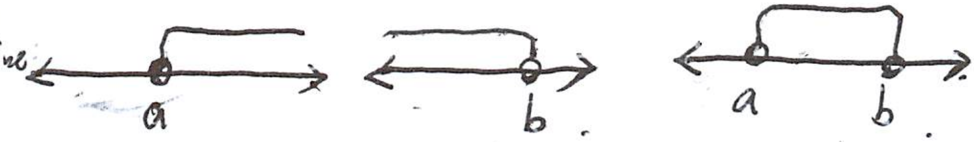
Prop. 9 ⇒ 〃 〃 〃 ⇔ $a > b$ or $a < -b$.

⑤ Interval notations.

<u>open interval</u>: $x > a$, $x < b$, $a < x < b$.

notation: (a, ∞). $(-\infty, b)$. (a, b).

on the number line



<u>closed interval</u>: $x \geq a$. $x \leq b$. $a \leq x \leq b$

notation. $[a, \infty)$ $(-\infty, b]$ $[a, b]$.

on the # line.

<u>Half open interval</u>: $[a, b)$. $(a, b]$.

<u>Real # line</u> $(-\infty, \infty)$.

Then distance between a, b now is . $|a-b|$.

④. Properties of absolute value. (Ref. Table 1.1 in the book)

1. $|a| \geq 0$.

2. $|-a| = |a|$

3. $|a|^2 = a^2$.

4. $|ab| = |a| \cdot |b|$. abs. val. of ~~product~~ prod = prod. of abs. val.

5. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$. $b \neq 0$. quotient = quotient

6. $-|a| \leq a \leq |a|$.

   b/c a is either $|a|$. (when $a \geq 0$) .

   or $-|a|$. (when $a < 0$) .

7. Let $b \geq 0$ $|a| = b \Leftrightarrow a = \pm b$. Very useful.

8. Let $b > 0$ $|a| < b \Leftrightarrow -b < a < b$.

9. Let $b > 0$, $|a| > b \Leftrightarrow a > b$ or $a < -b$.

10. $|a+b| \leq |a| + |b|$ triangle inequality.

Attendence Quiz

1. Solve $|5-3t| = 14$.

2. Solve $|x-8| \leq 1$.

Present your solution with the interval notations.

Example: Solve $|2x-6| = x$. (P18. EX 1).

Recall: $|a| = b$ ~~if~~ $\Leftrightarrow$ $a=b$ or $a=-b$.

b shall be required ≥ 0.

From the problem, $x \geq 0$.

$$|2x-6| = x \Leftrightarrow (2x-6 = x \text{ or } 2x-6=-x) \overset{\text{and}}{,} x \geq 0$$

$$\Leftrightarrow (x=6 \text{ or } x=2) \ \& \ x \geq 0.$$

So sol'n is ~~x=6~~, $x_1=6$, $x_2=2$.

Example: Solve $|2x-3| \leq 4$. (P19 EX3).

$$|2x-3| \leq 4 \Leftrightarrow -4 \leq 2x-3 \leq 4$$

$$\Leftrightarrow -1 \leq 2x \leq 7 \Leftrightarrow -0.5 \leq x \leq 3.5$$

Example: Solve $|x+8| = |3x-4|$. (P18. EX 2).

Either $x+8 = 3x-4$ or $x+8 = -(3x-4)$.

$|x+8| = |3x-4| \Leftrightarrow x+8 = |3x-4|$. $x \geq -8$.

or $x+8 = -|3x-4|$ $x < -8$.

$\Leftrightarrow$ ($x+8 = 3x-4$ $x \geq -8$.

or $x+8 = -(3x-4)$ $x \geq -8$)

or ($3x-4 = -x-8$. $x < -8$.

or $3x-4 = x+8$. $x < -8$).

$\Leftrightarrow$ $x+8 = 3x-4$. or. $x+8 = -(3x-4)$.

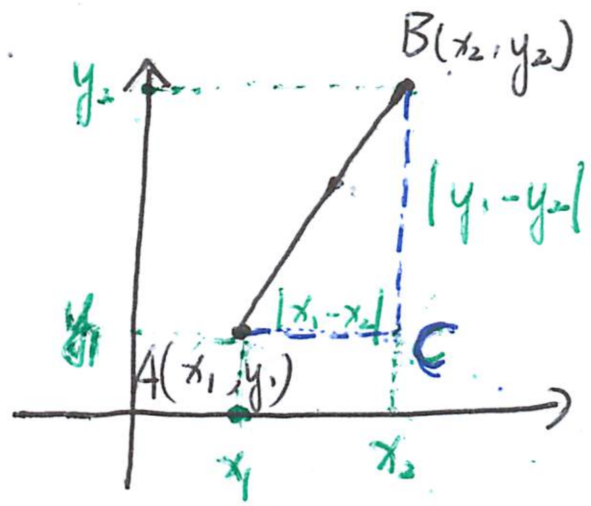
$x+8 = 3x-4 \Leftrightarrow 12 = 2x \Leftrightarrow x = 6$.

or $x+8 = -(3x-4) = -3x+4 \Leftrightarrow 4x = -4 \Leftrightarrow x = -1$.

⑤ Distance in the plane.

Distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$



$$d = \overline{AB} = \sqrt{\overline{AC}^2 + \overline{BC}^2}.$$

$$= \sqrt{|x_1 - x_2|^2 + |y_1 - y_2|^2}.$$

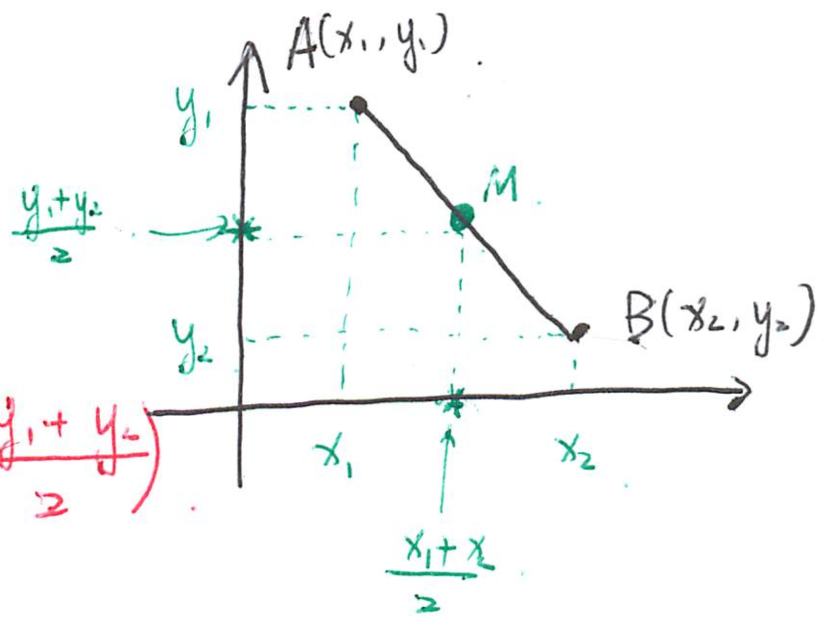
$$(|a|^2 = a^2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

⑥. Midpoint formula:

M midpoint of AB.

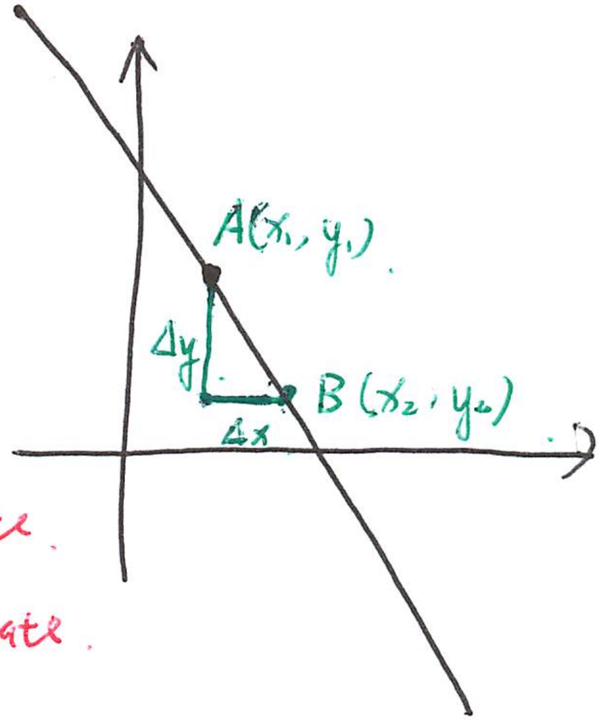
Coordinate of M?

<u>Ans</u>: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$



§ 1.3.

① Slope of a line.



$$m = \frac{\Delta y}{\Delta x}.$$

Δx change ~~of~~ of x-coordinate.

Δy change of y-coordinate.

If $A(x_1, y_1)$, $B(x_2, y_2)$. then

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Remark: If you know two points on the line, by the above formula you get the slope.

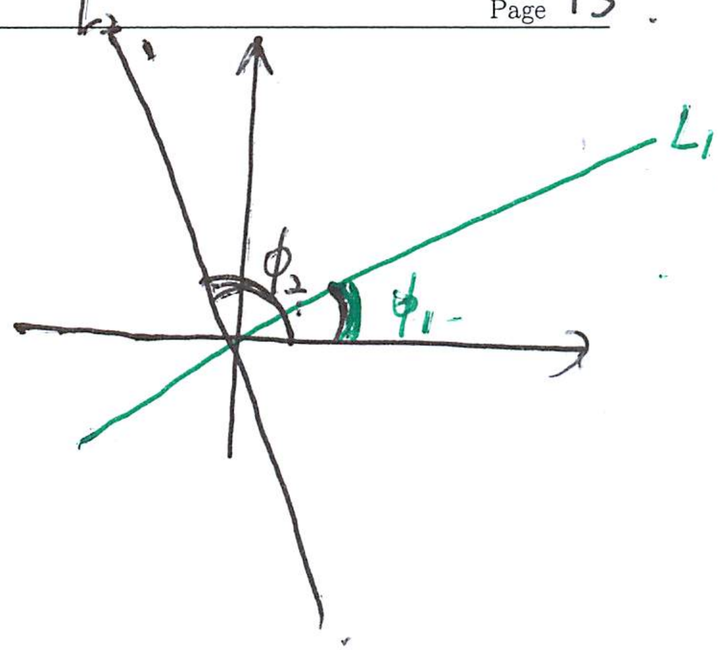
Exercise: Find the slope of the line that contains $A(2, 1)$, $B(0, 3)$.

Ans: $m = -1$.

② Angle of inclination.

$0 \le \phi < \pi$.

$m = \tan\phi$.



③ Equation of a line.

Standard form: $Ax + By + C = 0$.

Slope-intercept form: $y = mx + b$.

slope y-intercept

Point-slope form: $y - y_1 = m(x - x_1)$.

(x_1, y_1) any point on the line.

m slope.

Horizontal line: $y = y_0$. slope $= 0$.

Vertical line: $x = x_0$. No slope. (slope "$= \infty$")

Attendence Quiz:

3. Find the eqn of the line passing through (4,5), parallel to the line passing through (2,1) & (5,9).

4. Find the eqn. of the line ~~passing through (3, 2), perpendicular to 4x-3y+2=0~~ passing through (-1, 7) and (-2, 9).

3: $m = \frac{9-1}{5-2} = \frac{8}{3}$.

$y-5 = \frac{8}{3}(x-4)$ ✓

$y = \frac{8}{3}x + b$. $5 = \frac{8}{3} \times 4 + b$.

$b = 5 - \frac{32}{3} = \frac{5}{1} - \frac{32}{3} = \frac{15-32}{3} = -\frac{17}{3}$.

$\frac{32}{3} = 10\frac{2}{3}$ not recommended

$\Rightarrow y = \frac{8}{3}x - \frac{17}{3}$ ✓

Digression: $\frac{1}{3} = 0.3333\cdots$

$1 = 3 \times \frac{1}{3} = 0.9999\cdots$

If a ~~real~~ nonnegative # is < all positive #s, then it must be 0. (not easy) Math 311 411.

$1 - 0.9999\cdots$ is a nonnegative #

it is strictly less than any positive #.

e.g. 0.001 is larger than the difference

ANY POS. # > the difference ⇒ the difference = 0.