Quiz 6

MATH 251, MULTIVARIABLE CALCULUS

Problem 1: Find the critical points of the function $f(x, y) = x^2 + y^2$ subject to the constrain $2x + 3y = 6$

Let $g(x, y) = 2x + 3y - 6$. $\nabla f = \langle 2x, 2y \rangle$; $\nabla g = \langle 2, 3 \rangle$. The critical points are the solutions of the system

$$\begin{cases} 2x = 2\lambda \\ 2y = 3\lambda \\ 2x + 3y = 6 \end{cases}$$

We can solve for λ in the first two equations, and find $3x = 2y$, which together with the last equation, gives

$$x = \frac{12}{13},\ y = \frac{18}{13}$$

Problem 2: Evaluate the following double integral on the region $\mathcal{A} = [0, 2] \times [1, 2]$:

$$\iint_{\mathcal{A}} \frac{1}{(x + 3y)^2} dA$$

We can use Fubini's theorem and rewrite as iterated integral:

$$\begin{aligned} \int_0^2 \int_1^2 \frac{1}{(x + 3y)^2} dy dx &= \int_0^2 \left[\frac{-1}{3(x + 3y)} \right]_{y=1}^{2} dx \\ &= \frac{1}{3} \left(\int_0^2 \frac{1}{x + 3} dx - \int_0^2 \frac{1}{x + 6} dx \right) \\ &= \frac{1}{3} \Big[\ln(x + 3) - \ln(x + 6) \Big]_0^2 \\ &= \frac{1}{3} \left(\ln\left(\frac{5}{8}\right) - \ln\left(\frac{1}{2}\right) \right) \\ &= \frac{1}{3} \ln\left(\frac{5}{4}\right) \end{aligned}$$