Quiz 5

Problem 1. Find the critical points of the function

$$f(x,y) = x^3 - 3xy + y^3$$

determine whether they are local maxima, local minima, neither of them, or if the test is inconclusive.

Start by computing the gradient of f: $\nabla f = \langle 3x^2 - 3y, 3y^2 - 3x \rangle$; find critical points by setting the gradient equal to zero. We get a system of two equations

$$\begin{cases} 3x^2 - 3y = 0 \\ 3y^2 - 3x = 0 \end{cases}$$

with solutions $(0,0)$ and $(1,1)$.

The discriminant

$$D = \begin{vmatrix} \text{6x} & \text{-3} \\ \text{-3} & \text{6y} \end{vmatrix} = 36xy - 9$$

is positive at $(1,1)$ and negative at $(0,0)$. So, $(0,0)$ is a saddle point and $(1,1)$ is a local extremum. Since $f_{xx}(1,1) = 6 > 0$, $(1,1)$ is a local minimum.

Problem 2. Let $y(x,z)$ be implicitly defined by the equation $x^2y + y^2z + xz^2 = 10$. Calculate the partial derivative $\frac{\partial y}{\partial z}$

Let $F(x,y,z) = x^2y + y^2z + xz^2 - 10$; then

$$\frac{\partial y}{\partial z} = -\frac{F_z}{F_y} = -\frac{y^2 + 2xy}{x^2 + 2yz}$$