Quiz 4

MATH 251, MULTIVARIABLE CALCULUS

SHOW ALL YOUR WORK.

Problem 1: Find the directional derivative of $f(x, y) = \sin(x - y)$ in the direction $v = \langle 1, 1 \rangle$ at the point $P = (\pi/2, \pi/6)$.

$\nabla f = \langle \cos(x - y), -\cos(x - y) \rangle$

$\nabla f(\pi/2, \pi/6) = \langle \cos(\pi/2 - \pi/6), -\cos(\pi/2 - \pi/6) \rangle = \langle \cos(\pi/3), -\cos(\pi/3) \rangle = \langle \frac{1}{2}, -\frac{1}{2} \rangle$

$D_v f(\pi/2, \pi/6) = \langle \frac{1}{2}, -\frac{1}{2} \rangle \cdot \frac{1}{\sqrt{2}} \langle 1, 1 \rangle = 0$

Problem 2: Find a vector normal to the surface $x^2 + 3xy - y^2 + xyz = 5$ at the point $(1, 1, 2)$.

The surface is the zero set of the function $f(x, y, z) = x^2 + 3xy - y^2 + xyz - 5$; the normal direction is given by the value of the gradient at the given point.

$\nabla f = \langle 2x + 3y + yz, 3x - 2y + xz, xy \rangle$

so $n = \nabla f(1, 1, 2) = \langle 7, 3, 1 \rangle$