Quiz 2

MATH 251, MULTIVARIABLE CALCULUS

Problem 1
Let $P = (3, 0, 1)$, $Q = (1, 1, 0)$, $R = (2, 1, -1)$. Find an equation of the plane passing through P, Q and R.

Let $v = Q - P = \langle -2, 1, -1 \rangle$ and $w = R - P = \langle -1, 1, -2 \rangle$

$$n = v \times w = \begin{vmatrix} i & j & k \\ -2 & 1 & -1 \\ -1 & 1 & -2 \end{vmatrix} = \langle -1, -3, -1 \rangle$$

So the equation is $-(x-3) - 3y - (z-1) = 0$ or $x + 3y + z = 4$

Problem 2 Do the line $r(t) = (1, 1, 1) + t\langle 2, -1, 1 \rangle$ and the plane $x - 2y + z = 1$ intersect? If they do, find the point of intersection.

$r(t) = (1 + 2t, 1 - t, 1 + t)$

Plug into the equation of the plane:
$1 + 2t - 2(1 - t) + 1 - t = 1$

Simplify and solve for t, we get
$t = 1/3$

Thus the line intersects the plane when $t = 1/3$, that is at the point
$r(1/3) = (1 + 2/3, 1 - 1/3, 1 + 1/3) = (5/3, 2/3, 4/3)$.

Problem 3 Find an equation for the tangent line to the curve

$$r(t) = \langle t^2, \cos(\pi t), \ln(t) \rangle$$

at $t = 1$.

$r(1) = (1, -1, 0)$

$r'(t) = \langle 2t, -\pi \sin(\pi t), \frac{1}{t} \rangle$

$r'(1) = \langle 2, 0, 1 \rangle$

So the equation of the tangent line is

$$L(t) = r(1) + tr'(1) = (1, -1, 0) + t\langle 2, 0, 1 \rangle$$