Quiz 1

MATH 251, MULTIVARIABLE CALCULUS

Problem 1
Let $v = \langle 3, 5, 1\rangle$ and $w = \langle 1, -1, 0\rangle$

a) Evaluate the dot product of v and w

b) Find the projection $v_{||}$ of v on w

Solution

a)

$$\begin{aligned} v \cdot w &= \langle 3, 5, 1\rangle \cdot \langle 1, -1, 0\rangle \\ &= 3 \cdot 1 + 5 \cdot (-1) + 1 \cdot 0 \\ &= 3 - 5 = -2 \end{aligned}$$

b)

$$v_{||} = \frac{v \cdot w}{w \cdot w} w = \frac{-2}{2}\langle 1, -1, 0\rangle \ = \langle -1, 1, 0\rangle$$

Problem 2 Calculate the cross product

$$(\mathbf{a} - 2\mathbf{b}) \times (3\mathbf{a} + 2\mathbf{b})$$

assuming $\mathbf{a} \times \mathbf{b} = \langle 1, 0, 2\rangle$

Solution

$$\begin{aligned} (\mathbf{a} - 2\mathbf{b}) \times (3\mathbf{a} + 2\mathbf{b}) &= \mathbf{a} \times 3\mathbf{a} + \mathbf{a} \times 2\mathbf{b} - 2\mathbf{b} \times 3\mathbf{a} - 2\mathbf{b} \times 2\mathbf{b} \\ &= 3(\mathbf{a} \times \mathbf{a}) + 2(\mathbf{a} \times \mathbf{b}) - 6(\mathbf{b} \times \mathbf{a}) - 4(\mathbf{b} \times \mathbf{b}) \\ &= 2(\mathbf{a} \times \mathbf{b}) + 6(\mathbf{a} \times \mathbf{b}) \quad (\text{since } \mathbf{a} \times \mathbf{a} = 0 = \mathbf{b} \times \mathbf{b} \text{ and } \mathbf{b} \times \mathbf{a} = -\mathbf{a} \times \mathbf{b}) \\ &= 8(\mathbf{a} \times \mathbf{b}) \\ &= 8\langle 1, 0, 2\rangle = \langle 8, 0, 16\rangle \end{aligned}$$