ALGEBRAIC TOPOLOGY - MATH 421

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Course description

Traditionally this has been a course on algebraic topology only, beginning with a detailed review of point-set topology. This year, I plan to introduce both algebraic and differential topology, as this approach seems to allow you delve more quickly into applications in modern algebraic and differential geometry and topology, as well as meet the needs of theoretical physics and more recent application areas, such as economics.

Please feel free to contact me if you would like certain topics included which I have not listed or are unsure about your mathematical preparedness for the course. We may not cover all of the topics listed below, but we will try to cover as many as we can with care.

While the course consists of two "halves", we will move back and forth between the algebraic and differential viewpoints and compare methods as needed. At the end of this information sheet, you will find a list of references: most, but not all, of these books will be added to the Hamilton library shortly.

Algebraic topology. The principal source for this half of the course will be the book by Greenberg and Harper [10]: it requires very minimal background, is fairly short and accessible, and leads directly to quite advanced topics. The book by Massey [14] is also good and Fulton [9] contains a wealth of examples. If you need to review point-set topology (and fundamental groups), then the book by Munkres [17] is a good place to start.

- Quick review of point-set topology.
- Elementary homotopy theory. Fundamental groups. Covering spaces.
- Singular homology theory. Exact sequences. Excision. Mayer-Vietoris sequence. Cohomology theory. Duality. Cup and cap products.

Differential topology. There is no one book which is ideal, but we will rely most on the books by Guillemin and Pollack [11], Milnor [16], [15], and Warner [25].

- Differential manifolds and differential maps, tangent bundles, implicit function theorem, immersions, submersions, and embeddings, submanifolds. Vector bundles. Normal bundles and tubular neighborhoods.

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Transversality and Sard's theorem. Abstract manifolds and Whitney embedding theorems. Morse theory.
- Manifolds with boundary. Mod 2 intersection theory. Applications.
- Orientation. Oriented intersection theory. Applications.
- Integration on manifolds, differential forms and De Rham cohomology. Poincaré duality.
- Symplectic manifolds.

Course details

Duration. Full year.

Number of lectures per week. 3.

Prerequisites. Ideally, you should have taken Mathematics 212 (Topology). It will also be helpful to have had Mathematics 211 (Linear algebra and differential forms) and Mathematics 221 (Real and complex analysis). Depending on the class background, we will quickly review point-set topology and vary the pace as needed.

Assessment

- Lists of practice problems will be assigned every week or every other week. These do not contribute to the final grade, but should lead to a thorough preparation for the final exam. You are encouraged to work together in groups on these assignments. The main thing is to try as many problems as you have time for.
- One 3-hour end-of-year examination.

References

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[24] C. T. Wall, *A geometric introduction to topology*, Addison-Wesley, London, 1972.
[25] F. W. Warner, *Foundations of differentiable manifolds and Lie groups*, Springer.

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