Math 151 Workshop Problems, Fall 2001, Week 12

1. Consider the functions given by the equation $f_c(x) = (x^2 + c)e^x$, where c is a parameter.

(a) Using the calculator, observe the curves for $-4 \leq x \leq 1$, for the values $c = 0, 1, 2$. Do all three curves have the same number of horizontal tangents? the same number of inflection points? You may have to ZOOM in to investigate this.

(b) Use calculus to determine the location of all the inflection points of the graph of $y = f_c(x)$. Your answer will of course depend on c.

(c) At what values of c does the number of inflection points change? What are the values of c (if any) for which there is exactly one inflection point?

2. Imagine using the linearization of $\arccos x$ at $x_0 = 1/2$ to calculate an approximate value for $\arccos(.49)$.

(a) What do the first and second derivatives tell you about the graph of $\arccos x$ near $x = 1/2$?

(b) Use your answer to (a) to decide if the approximate value calculated by linearization is an overestimate or an underestimate of the true value.

(c) Now do the linearization and compare the approximate value with the true value to check your answer to (b).

3. A piece of wire $18'$ long is to be cut into two pieces, one to be bent into a square and the other to be bent into an equilateral triangle. Let A be the sum of the areas of the square and the triangle.

(a) How should the wire be cut to maximize A?

(b) How should the wire be cut to minimize A?

(c) How would your answers differ if it were permitted not to cut the wire, but to use the whole wire for a square or triangle?

4. If f is a function, then a real number x_0 is called a *fixed point* of f if and only if $f(x_0) = x_0$.

(a) Find all the fixed points of the following functions. Explain your answers either algebraically or graphically.

$$f(x) = x^2 \qquad g(x) = x^5 \qquad h(x) = \frac{2}{3}\arctan x.$$

(b) Suppose that f is a differentiable function and $f'(x) < 1$ for all x. Use the MVT to explain why f can have no more than one fixed point. To which of the functions in (a) does this general statement apply?

5. You are $6'$ tall. A $5'$-high painting is mounted on a wall so that its lower edge is $1'$ above your eye when you stand next to the wall.

a) Suppose that you stand x feet away from the wall to look at the painting. Find a formula for your viewing angle θ between the top and bottom of the painting, as a function of x. (*Hint:* Draw a horizontal line at eye level.)

b) Graph θ as a function of x and estimate from the graph where it has a maximum. Now use calculus to find the value of x that maximizes θ.

6. A charged particle moves along the x-axis under the influence of an electric field. The field strength varies with time, and as a result the velocity of the particle seems complicated. Let the position of the particle at time t be denoted by $x = x(t)$, and the velocity of the particle at time t be denoted $v = v(t)$.
Suppose that $x(0) = 0$, and

$$v(t) = \begin{cases} 8t - 2, & \text{if } 0 \le t \le 1 \\ 9t - 3t^2, & \text{if } 1 \le t \le 2 \\ 8 - t, & \text{if } 2 \le t \le 3 \end{cases}$$

Sketch the graph of $v = v(t)$. What is $x(1)$? What is $x(2)$? What is $x(3)$? Sketch the graph of $x = x(t)$.

7. Find a function $f(x)$ such that $f''(x) = e^{-x}$ and the graph of $y = f(x)$ passes through the point $(0, 3)$ and has a horizontal tangent there. Do these conditions determine f completely?

8. a) A car is traveling at 50 mi/h when the brakes are fully applied, producing a constant deceleration of 40 ft/s^2. What is the distance covered before the car comes to a stop?

b) A car braked with a constant deceleration of 40 ft/s^2 and produced skid marks measuring 160 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

9. (a) Draw the trapezoid with vertices $(2, 0)$, $(4, 0)$, $(2, 5)$ and $(4, 11)$, compute its area A, and find an equation for the top edge.

(b) Draw a figure and explain geometrically why the number

$$\frac{1}{100} \sum_{n=1}^{200} \left[3\left(2 + \frac{n}{100}\right) + 1 \right]$$

represents an area which is just slightly larger than A.

(c) Move the "error" pieces in your figure around to show that the difference between the two areas is no more than 0.06 square units.