Name: ______________________________

Math 151 Workshop Problems, Fall 2000, Week 2

1. Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{100 - x}$. Find the domain and range of each of the following functions and justify your assertions.

$$F(x) = f(g(x)) \qquad G(x) = g(f(x)) \qquad H(x) = \frac{f(x)}{g(x)}$$

$$K(x) = f(x) - g(x) \qquad L(x) = f(x) + \frac{1}{g(x)}$$

2. A tasty bug is at point $(x, 0)$ on the x-axis and is being watched by a sparrow perched at the point $(4, 9)$ and a woodpecker who is at the point $(-2, 5)$.

a) Find a formula for the distance from the sparrow to the bug, and a formula for the distance from the woodpecker to the bug.

b) For what values of x is the bug nearer the sparrow? Give an algebraic solution and a graphical solution.

3. (a) Graph the function $y = 3^x$ in the window $-1 \leq x \leq 1$. On the same axes draw the secant lines through the points $(x, 3^x)$ and $(0, 1)$ for $x = 0.5$ and $x = 0.1$.

(b) Let $m(x) = (3^x - 1)/x$ be the slope of the secant line to the graph in (a). Graph $m(x)$ for $-1 \leq x < 0$ and $0 < x \leq 1$. Give a table of values of $m(x)$ for $x = -.02, -.01, .01, .02$. (Use the TABLE key for the TI-82 or the EVAL key for the TI-85.)

(c) Determine the limiting value of $m(x)$ as x approaches 0, correct to five decimal places, by using a smaller ΔTbl in the table setup. Later in the term we will show that the limit is $\ln 3$. Check this numerically.

(d) Draw the secant line through $(x, 3^x)$ and $(0, 1)$ for $x = 0.0001$.

(e) What is the equation of the tangent line to $y = 3^x$ at $(0, 1)$?

4. Let $f_n(x) = (x^n)2^{-x}$.

a) Find the graphs of $f_n(x)$ for $0 \leq x \leq 10$ and $n = 1, 2, 3$. You will have to adjust the viewing window to see the graph. Describe how the graphs change as n increases. What features stay the same?

b) Find the x coordinate $x_{\max}$ of the highest point of the graph for $n = 1, 2, 3$. Plot $x_{\max}$ as a function of n.

c) Guess what the graph of $f_5(x)$ looks like, and what the x coordinate of the highest point is, based on the evidence from (a) and (b). Then test your guess by actually generating the graph.

5. Let $f(x) = 3^x$, $g(x) = x^3$, $F(x) = \log_3(x)$ and $G(x) = \sqrt[3]{x}$.

(a) Express each of the following six functions in as simple (but correct!) a form as possible, and find its domain and range:

$$f \circ g, \qquad g \circ f, \qquad f \circ G, \qquad g \circ F, \qquad F \circ G, \qquad G \circ F.$$

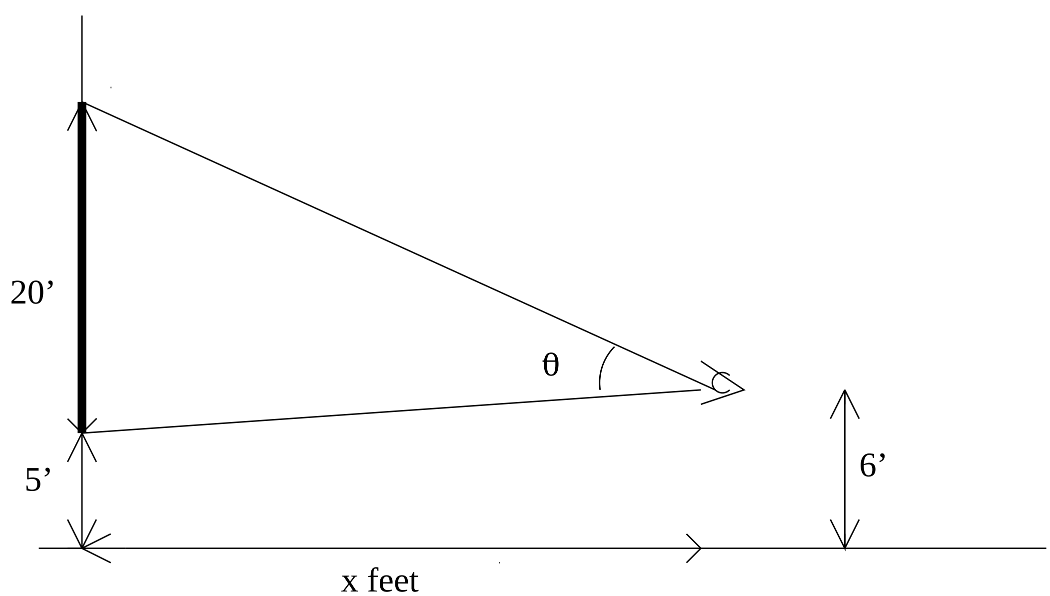
(b) The functions f and F are inverses of each other. (The meaning of this statement is that the equations $y = f(x)$ and $x = F(y)$ are equivalent.) So are g and G. Which of the above six functions is the inverse of $f \circ g$? Verify your assertion by checking that the equations $y = f \circ g(x)$ and $x = h(y)$ are equivalent, where h is your choice for the inverse.

(c) Find as many pairs of inverse functions as you can among the six functions above, and verify each pair in a similar way.

6. For each of the following equations, (a) say how many solutions there are in the interval $0 \le x \le \pi$; (b) find them all numerically, using a graph of the function $f(x) = 4^{\sin 2x}$, (c) obtain exact expressions for all the solutions satisfying $0 \le x \le \pi$, in terms of the log and arcsin (or $\sin^{-1}$) functions, and constants such as π; (d) check on the calculator that your answers to (b) and (c) coincide.

$$2 \cdot 4^{\sin 2x} = 1 \qquad 3 \cdot 4^{\sin 2x} = 1.$$

7. A large mural is $20'$ high and its bottom edge is $5'$ above floor level. My eye is $6'$ above the floor. Express my angle of view θ in terms of x, if I am standing x feet from the mural. (Hint: To get started, draw a horizontal line from my eye to the mural.)



8. After a softball game, the winning pitcher throws the ball straight up in the air. The height s of the ball in feet is given by the formula $s = 5 + 48t - 16t^2$, where t is the time after release (measured in seconds).

(a) The formula is valid only until the ball hits the ground. When does that happen?

(b) Find the instantaneous velocity of the ball at the following instants: $t = 1$, $t = 2$ and $t = 3$.

(c) When do you think that the ball reaches its highest point? Check your guess by computing the instantaneous velocity at that instant. How high does the ball get?