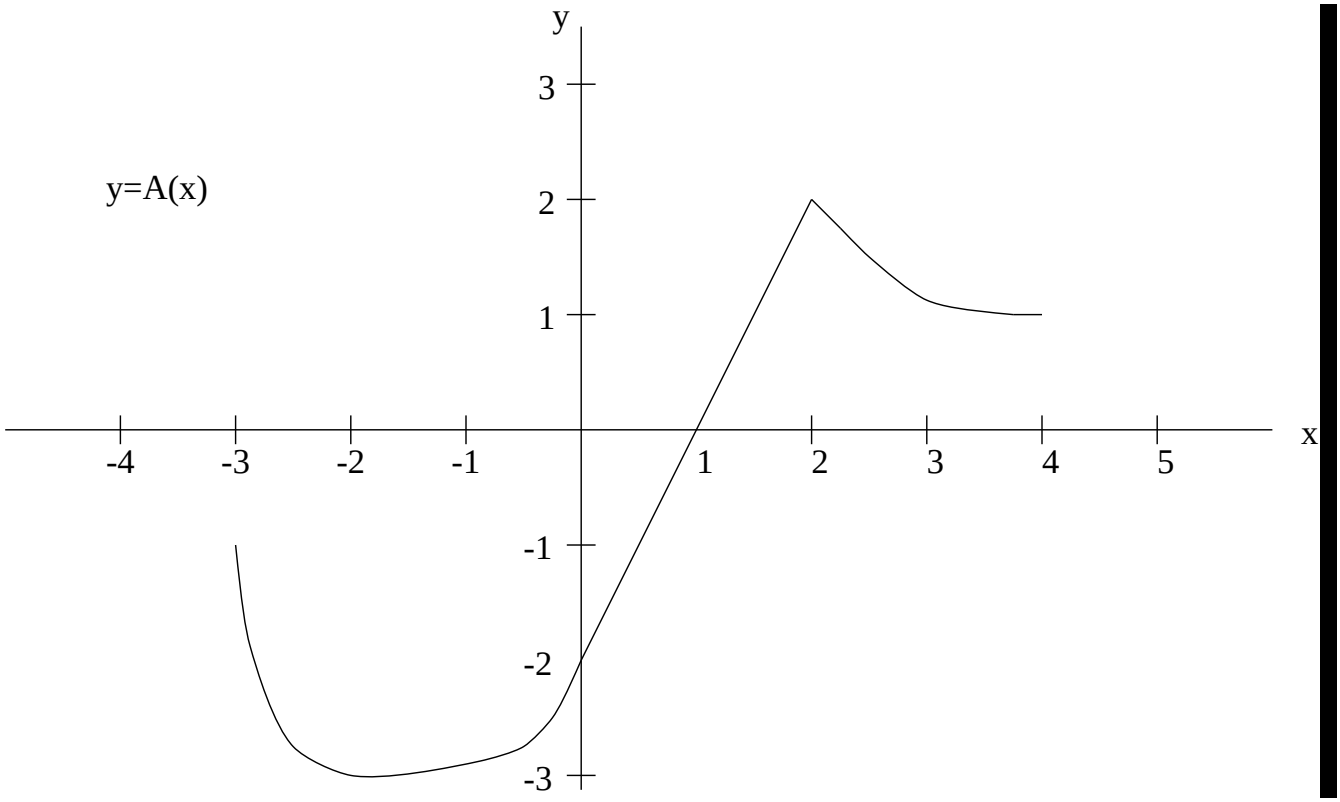
Name: _____

Calculus 151 Problems, Fall 2001, Week 1

Please write solutions to ONE of these problems (as announced in the Workshop) and hand it in at the Workshop next week. Your written solution should follow the guidelines in the sheet 'Writing up workshop problems' that you received at the workshop. You may discuss the problems with other students and with your instructors, but the written work you hand in must be your own. You should use your graphing calculator as an experimental tool whenever possible. For example, you can check particular cases of inequalities, or compare the graphs of functions this way.

DO NOT TRY TO WORK OUT THE SOLUTIONS ON THIS PIECE OF PAPER. Bring a notebook to workshop to record your calculations during the workshop. These notes can then be the starting-point (but not the finishing point) for the writeup that you hand in.

1. Here is a graph of $y = A(x)$. Answer without using a graphing calculator*.



(a) What are the domain and range of A?

(b) Suppose B is the function defined by $B(x) = 2A(x)$. Sketch the graph of B as well as you can. What are the domain and range of B? How is the graph of B related to that of A?

(c) Do the same as (b), but for the function C defined by $C(x) = A(2x)$.

(d) Do the same for the function D defined by $D(x) = A(2x - 3)$.

(e) Do the same for the function E defined by $E(x) = 2A(x) - 3$.

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* It's not possible to use your calculator anyway since you don't have a formula for $A(x)$.

2. Let $f(x) = \cos 20x$. Sketch the graphs of the following three functions for $x \geq 0$:

$$f(x), \qquad f(x^2), \quad \text{and} \quad f(\sqrt{x})$$

Be careful about getting sensible sketches from your calculator. Which of the three has the most high points and the fewest high points on the following intervals? $[0, .25]$, $[0, .5]$, $[0, 1]$, $[0, 2]$, $[0, 3]$. Explain how the high points are spread out differently for these three functions.

3. A piece of wire 180 inches long is bent into the shape of an isosceles trapezoid whose base angles are $\pi/3$ radians.

(a) Let x be the length of the longer base of the trapezoid and y the length of one of the slanted sides. Label the lengths of all sides in terms of x and y and deduce a relationship between x and y.

(b) Find a formula for the area A of the trapezoid as a function of the single variable x.

(c) Use your calculator to graph the function $A = A(x)$. Are there any upper or lower bounds between which the value of x must lie? If so, decide what happens to A as x approaches those limits, and explain by drawing pictures of the trapezoid in those cases.

4. a) Graph $y = |x - 2|$ and $y = |x - 3|$ on the same set of axes. From this graph, decide for which real numbers x the inequality $|x - 2| < |x - 3|$ is valid.

b) Interpreting $|a-b|$ as the distance between a and b on the number line, give a geometrical explanation why your solution in a) is correct.

c) Repeat for the inequality $|x + 1| < 2|x - 1|$.

d) The inequality $x+1 < 2(x-1)$ has a different solution (what is it?) from the inequality in c). Explain algebraically why these two inequalities cannot be expected to have the same solutions.