Math 151 & 153 final exam review problems

The final exam will cover the whole course: all the sections listed on the syllabus starting from Chapter 1, including the appendices. The following 11 problems emphasize material in Chapters 4 and 5 and Section 6.1. THE EXAM WILL ALSO HAVE PLENTY OF QUESTIONS FROM EARLIER MATERIAL. A complete review should certainly include the review problems for the midterm exams and those exams themselves.

1. Find the following derivatives, antiderivatives, and integrals:

a) Find $F(x)$ if $F'(x) = (2x+1)^{14}$ and $F(-1) = 2$.

b) $\int x^2 \sin(x^3)\, dx$

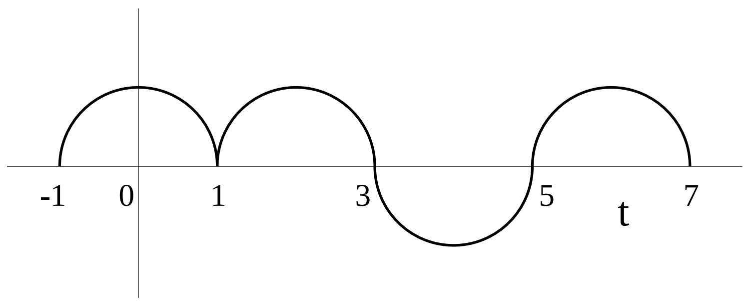
c) $\int_1^3 (x^2 - \sqrt{2x+1})\, dx$

d) $\int xe^{-2x^2}\, dx$

e) $\int_0^1 \frac{1+x}{1+x^2}\, dx$ (Hint: split the fraction in two.)

f) $\frac{d}{dx}\left(\int_0^{\sin x} \sqrt{1+t^4}\, dt\right)$

2. Below is a graph of $y = f(t)$:



Graph of $y = f(t)$

Each segment of the graph is a semicircle.

Define the function g by the formula $g(x) = \int_{-1}^{x} f(t)\, dt$.

a) Use the signed area definition of the definite integral to calculate $g(-1)$, $g(0)$ and $g(5)$ (no calculus needed).

b) In what intervals is g increasing? In what intervals is g decreasing?

c) Sketch a graph of $y = g(x)$ for $-1 \le x \le 7$.

3. In this problem, $f(x) = \sqrt{x}$.

a) Write out the Riemann sum

$$\sum_{i=1}^{4} f(x_i^*)\, \Delta x_i$$

for the partition $\{0, .5, 1, 1.5, 2\}$ of the interval $[0, 2]$ and the evaluation points x_i^* = right endpoint. (you do not have to give a decimal value for the sum).

b) Express the following limit as a definite integral, and then evaluate the integral by calculus:

$$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{2i}{n}\right) \frac{2}{n}$$

4. The angle of elevation of the sun is decreasing at a rate of 0.26 radian per hour. How fast is the shadow cast by a 400 foot tall building increasing when the angle of elevation of the sun is $\pi/6$?

5. A vendor of Italian ices makes containers by cutting circular disks of paper along a radius and rolling them into the shape of a cone. If the radius of the disk is 6", what is the volume of the largest cone that can be made? Justify your answer. (*Note:* The volume of a right circular cone of radius r and height h is given by $V = \frac{\pi}{3}r^2h$.)

6. Suppose you know that $f(2) = 3$ and $f'(x) = e^{-x^2}$ (there is no simple formula for $f(x)$ itself). Find the function $L(x)$ that is the linear approximation to $f(x)$ around the point $x = 2$. Do you expect that $L(2.1)$ is *larger* or *smaller* than $f(2.1)$? Give a reason for your answer based on the concavity of the curve $y = f(x)$ near $x = 2$.

7. A falling raindrop has a velocity $v_0 = 10$ meters/sec at time $t = 0$. Assume that the downward acceleration on the raindrop at time t is $9 - 0.9t$ when $0 \le t \le 10$ and that the acceleration becomes 0 for $t > 10$, due to wind resistance. Assume the raindrop is 500 meters above the ground at time $t = 0$.

a) What is the position and velocity of the raindrop at time $t = 10$ seconds?

b) At what time does the raindrop reach the ground?

8. Sketch the graph of $f(x) = \ln x + \dfrac{2}{x}$, $x > 0$. Use calculus to determine the location of all vertical and horizontal asymptotes, local extrema and points of inflection. How does the graph of f compare with that of $\ln x$ for large x?

9. Suppose $F(x) = x^3 - x^2 - 8x + 1$ on the domain $[-2, 2]$. Use calculus to locate precisely the absolute maximum and absolute minimum of F on this domain. Similarly, find any local maxima and minima of F on this domain.

10. Consider the function J whose *derivative* is given by the following formula:

$$J'(x) = \frac{x-1}{x^2+x}$$

Answer the following questions as well as you can (you may find it helpful to refer to features of the graph of $y = J'(x)$ in what follows):

a) In what intervals is J increasing? In what intervals is J decreasing? Why?

b) Does J have any critical numbers? Where does J have local extrema, and what kind of extrema are they? Give two explanations: one using your answer to a), and the other using the Second Derivative Test.

c) Where are J's points of inflection located? What are the largest intervals in which J is concave up? What are the largest intervals in which J is concave down?

11. a) Let R be the finite region enclosed by the curves $y = \dfrac{8}{x}$ and $y = 6 - x$. Find the area of R.

b) Let A be the finite region enclosed by $y = 1/x$, the x-axis and the lines $x = 2$ and $x = 72$. Which vertical line divides A into two pieces of equal area?