16:640:549 - Lie Groups

Instructor: Paul Feehan (feehan@math.rutgers.edu)

Schedule: Fall 2018, Tuesdays, 10:20-11:40, Hill 425

Course Description:

Lie Groups are of central importance in Mathematics and required background for every research mathematician and theoretical physicist. At the nexus of Analysis, Geometry, and Topology, Lie Groups have essential applications to Algebraic and Differential Geometry, Differential Equations, Mathematical Physics, Topology, and many other areas of Mathematics and Physics. This course will give a broad introduction to Lie Groups, with minimal prerequisites. Topics may be modified depending on the audience and their interests but should include:

1. Classical matrix Lie groups and Lie algebras
2. Topological, smooth, and analytic manifolds
3. Group actions on manifolds
4. Vector fields and the exponential map
5. Peter-Weyl Theorem
6. Lie groups and Lie algebras
7. Baker-Campbell-Hausdorff formula
8. Structure and representation of complex semi-simple Lie algebras and Lie Groups
9. Fundamental group of a Lie group
10. Principle bundles
11. Morse theory and Bott Periodicity Theorem

Prerequisites: Real Analysis, Linear Algebra, and Elementary Topology or permission of the instructor.

Primary References:

No one textbook will suit every student or be a perfect reference for the entire course, but the course notes will draw on the following excellent primary references.

- T. Bröcker and T. tom Dieck, *Representations of compact Lie groups*, Springer, 1985.
- S. Helgason, *Differential geometry, Lie groups, and symmetric spaces*, American Mathematical Society, 2001.
- J. Hilgert and K-H. Neeb, *Structure and geometry of Lie groups*, Springer 2012.
- V. S. Varadarajan, *Lie groups, Lie algebras, and their representations*, Springer, 1984.

Secondary References:

The following references will be useful for students whose interests tend more towards more specific applications, for example in Algebra and Number Theory or in Differential Geometry and Topology.

- B. A. Dubrovin, A. T. Fomenko, and S. P. Novikov, S. P., *Modern geometry – methods and applications*, Parts I, II, and III, Springer, 1985, 1990, and 1992.
- W. Fulton and J. Harris, *Representation theory – A first course*, Springer, 2004.
- S. Kobayahsi and K. Nomizu, *Foundations of differential geometry*, Vols. 1 and 2, Wiley, 1996.

- J. W. Milnor, *Morse theory*, Princeton University Press, 1963

Other References:

- D. Bump, *Lie groups*, Springer, 2013.
- J. J. Duistermaat and J. A. C. Kolk, *Lie groups*, Springer, 2000
- R. Godement, *Introduction to the theory of Lie groups*, Springer, 2017
- B. Hall, *Lie groups, Lie algebras, and representations*, Springer, 2015, Second edition
- J. Humphreys, *Introduction to Lie algebras and representation theory*, Springer, 1972
- J. Knapp, *Representation theory of semisimple groups - An overview based on examples*, Springer, 1986
- C. Procesi, *Lie groups*, Springer, 2007
- M. Sepanski, *Compact Lie groups*, Springer, 2007
- F. Warner, *Foundations of differentiable manifolds and Lie groups*, Springer, 1983.