MATH 574 ASSIGNMENT 2

1. Consider the tridiagonal matrix

$$A = \begin{pmatrix} a_1 & c_1 & 0 & & & \dots & 0 \\ b_2 & a_2 & c_2 & 0 & & \dots & 0 \\ 0 & b_3 & a_3 & c_3 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & & \vdots \\ 0 & \dots & & b_{n-1} & a_{n-1} & & c_{n-1} \\ 0 & \dots & & 0 & b_n & & a_n \end{pmatrix}.$$

If A is irreducible and weakly diagonally dominant, show that all a_i, b_i, and c_i are nonzero.

2. Show that if A is strictly diagonally dominant, then

$$\|A^{-1}\|_\infty \leq \frac{1}{\min_i |a_{ii}| \left[1 - \max_i \sum_{j \neq i} |a_{ij}/a_{ii}|\right]}.$$

Hint: Write $A = D[I + D^{-1}(L+U)]$ and use $\|(I+E)^{-1}\| \leq 1/(1 - \|E\|)$, for $\|E\| < 1$.

3. Let

$$A = \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix}$$

a) Find the iteration matrix M for the Jacobi method, Gauss Seidel method, and SOR method.

b) Compute the spectral radius in each case (for SOR, take $\omega = 4(2-\sqrt{3})$). Use exact arithmetic until the last step of your computation. Note that with this choice of ω, we have $\omega^2/8 - 2\omega + 2 = 0$.

4a. Find the values of the constant a for which the matrix

$$A = \begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix}$$

is positive definite.

Hint: Use the fact that a symmetric matrix is positive definite if and only if all its eigenvalues are > 0.

4b. Find a vector $x = (x_1, x_2)^T$ and a constant a *not* satisfying the conditions of part (a) for which $x^T A x \leq 0$.