

**THE FINITE ELEMENT  
“CIRCUS”**

This book is dedicated  
to the participants of the  
Finite Element Circus  
and in particular to  
Prof. Ivo Babuska for  
his inspiring leadership.

May F. Wheeler

May 3, 1977  
Circus

## Fall Meeting of the FINITE ELEMENT CIRCUS

The Finite Element Circus held its fall meeting at the University of Maryland on November 11, 12, 1977. Forty-seven people attended the meeting and Ivo Babuška was the coordinator and host. Nineteen presentations were given. On Friday evening, November 11, the group enjoyed dinner and entertainment at the Burn Brae Dinner Theater where a performance of "Carousel" was given.

<u>Babuška</u>	John Osborn
<u>William W. Hager</u>	Patricia M. Fitzpatrick
<u>RANzolaidis</u>	Michael Vogelius
<u>Klaus G. Kuper</u>	Dick Morgan
<u>Jakob Tikkonens</u>	Steve Cauchedi
<u>Charles Goldstein</u>	William Kalata
<u>Alan E. Berger</u>	Véronique Meyer
<u>Ridgway Scott</u>	Joe Pasciak
<u>Bernard Meurier</u>	Ragnar Winther

Polo Sammon

Richard S. Falk

J. Thomas King

Steven M. Leslie

Gerard Richter

Stephen Leventhal

Arthur G. Werschulz

Melvyn Ciment

Alfred Schatz

Jim Branbell

Albert K. Ozog

Robert G. Hunt

Zukan Yacko

Annie V. Taylor

Betsy Greenwell

Jeff E. Lewis

Jim Hall

Julian W.

David Archer

Mitchell Just

Richard Evans

Ernest B. Becker

Mary F. Wheeler

Graham F. Guy.

Bruce Kellogg

Peter Percell

J. Ansley Oden

Todd Dupont

Simpson, glass &

Linda G. Hayes

Greene Fairweather

Joel C. W. Rogers

Mitchell Luskin

Calculation of Ocean Tides and Currents

Bertrand Mercier

Equivalence of some finite element methods when applied to the 2-d. Stokes problem.

CHARLES GOLDSTEIN

A NUMERICAL STUDY OF THE WEAK ELEMENT METHOD APPLIED TO THE HELMHOLTZ EQUATION

Joe PASCIAK

THE PENALTY CORRECTION METHOD FOR ELLIPTIC BOUNDARY VALUE PROBLEMS

Art Werschulz, Kadir Aziz, Steve Leventhal

A Superconvergence Result for the Numerical Solution of the Tricomi Problem

Steve Leventhal, Alan Berger, M. CLEMENT, Say Solomon, B. Wren

Fourth Order O.C.I. methods

Bruce Kellogg - Alice Tsan

Uniform (in  $\epsilon$  and  $b$ ) error estimates for singular perturbation problems

BILL HAGER

APPROXIMATIONS TO CONVEX CONTROL PROBLEMS

Bill Kolata

Eigenvalue approximation by the method of Lagrange multipliers.

Jeff Lewis - An Elementary Proof of  $H^2$ -Regularity for Poisson's Equation.

R.A. Nicolaides — Ideas of multiquid methods

Mary F. Wheeler and (P. Percell) - Orthogonal Collocation for Elliptic Equations

P. Percell - Equivalence of two norms on  $M_1(\Omega, S)$  with constants independent of  $S$ .

R. Winther - Initial value methods for parabolic optimality systems.

J. Tinsley Oden - EXISTENCE THEOREMS AND FINITE ELEMENT APPROXIMATIONS OF BOUNDARY VALUE PROBLEMS IN NONLINEAR ELASTICITY

RICHARD EWING - INCOMPLETE ITERATIVE METHODS FOR TIME STEPPING INITIAL-BOUNDARY VALUE PROBLEMS FOR NONLINEAR PARABOLIC PROBLEMS

H. Brezis, J.C. W. Rogers, ArE. Berger - A Numerical Method Motivated by Nonlinear Semigroups for Solving the Generalized Stefan Problem  $u_t - \Delta f(u) = 0$

R. Scott - More Adventures of E-Man :  
Approximation Rates for Parabolic Problems  
in the Absence of Full Compatibility Conditions  
I. Babuska - A posteriori estimates and  
of course mushes

## Finite Element Circus Meeting

Austin, Texas Nov. 17, 18, 1978

The meeting opened at 9 AM at the Academic Center. It was attended by 34 people and 20 people signed up to speak. Each speaker was given a time allowance of 30 minutes. The morning session consisted of 5 lectures and then there was a break for lunch. We were the guests of TICOM at the Faculty Center.

After five more lectures in the afternoon the meeting ended for the day at 5 o'clock. The evening was a particularly pleasant one. A bus trip to Coupland, Texas was organized by Tinsley Oden. In Coupland we had a Texas barbecue in an old and famous establishment, after which we went for about an hour to the Coupland Tavern and listened to some Country-Western music. Everyone seemed to enjoy himself. In keeping with the contents of the talks, everyone had bullshit stamped on his hand.

The meeting started again at 9 AM and the remaining 10 lectures were presented during the day. It was announced that the spring 79 circus will be held in Ann Arbor, Michigan with Mitch Luskin and R.Dg. Scott as our hosts. Fall will see the circus back in Ithaca again. It seems that our field of interest is yet extremely dynamic and that the circus remains an excellent vehicle at which friends and colleagues can communicate.

Graham J. Conroy

Bethz	Jm Douglas
Mitchell Luskin	T. Bernasow
Mary Janett Whittle	H-Jürgen Reinhardt
Dinda Hayes	William D. Kolata
Tinsley Oden	John E. Osborn
Richard C. Ewing	Joe Kroone
G. Attaway	Noah Kikuchi
Peter J. Monner	Stephen Leventhal
A.K. Agar	Steven M. Seltin
	Richard S. Falk

Alan E. Berger  
Peter Percell  
Bruce Kellogg  
Jim Bramble  
Michael E. Noe  
Ragnar Winther  
Andy Sherman

Todd Dupont  
Dan B. Wahlbin  
Charles Goldstein  
Ridg Scott  
Bruce L. Darlow  
Randy Bank  
Eric B. Baker  
Graham F. Corry

there once was a fellow named Dare  
who approximated P.D.E.'s with great care.  
But the solutions were rough,  
And the problems were tough;  
so he never lay! begot than  $O(h^2)$ .

R. Falk

Titles

Mitchell Luskin - Approximation of the Spectral  
Properties of Closed Operators (Joint work  
with ~~J. Descloux~~ J. Descloux and J. Rappaz).

Mary Fanett Wheeler - Galerkin Methods for  
Miscible Displacement Problems in Porous  
Media (Joint work with R. Ewing)

Jerrold Douglas Jr - Superconvergence for the Galerkin  
Approximation to the Solution of the Benjamin-  
Bona-Mahoney Equation (Joint work with  
Douglas N. Arnold)

Richard E. Ewing - Efficient Time-stepping  
Methods for Miscible Displacement  
Problems in Porous Media

Michel Bernadou - Curved finite elements of  
class  $C^1$  and numerical integration for  
shell problems.

John E. Osborn - Remarks on the analysis of  
mixed methods for the Biharmonic equation  
(Joint work with I. Babuška and J. P. Bérenger)

A. K. Aziz - Remarks on error estimates for Helmholtz  
in the finite element method.

Steven M. Serrin - An Approximation Theorem  
for Second-Order Evolution Equations  
(joint work with Garth Baker and Vassilios Dougalis)

Richard S. Falk - Numerical Approximation of  
a Cauchy Problem for a Parabolic P.D.E.  
(joint work with Richard E. Ewing)

Alan E. Berger - Error Estimates for Generalized Operator  
Compact Implicit (OCI) Finite Difference Schemes  
for a Singular Perturbation Problem (joint work with  
Jay Solomon, Melvyn Ciment, Stephen Leventhal, & Bernard Weisberg)

Todd Dupont - Some preliminary remarks on the  
effect of singular source terms in incompressible  
displacement problems.

Lars R. Wahlbin, A weak maximum principle in  
parabolic finite element discretisations.

Bruce Kellogg - mesh refinements & weighted  
Besov spaces (joint with I. Babuska & J. Pitkäranta)

Jim Brinkley (with Peter Sammon) Efficient  
higher order time stepping schemes for  
time dependent and quasi-linear parabolic  
problems.

Ridgway Scott (and Todd Dupont) -

Polynomial Approximation in Degenerate Domains

Michael Rose

Numerical Methods for  
Degenerate Parabolic Equations.

Bandy Bank (+ Todd Dupont) - Multi-level  
methods for parabolic eqns.

Ragnar Winther: Modified Galerkin methods for the  
Korteweg-deVries equation.

Graham Carey (and Kamay Sepahnoori) -

'Gerschgorin theory for Time Dependent Systems'

Ivo Babuška

Tools of convergence for the  $\delta$ -version  
of the finite element method

Finite Element Circus Meeting  
Maryland, College Park May 18-19-1979

The Texas meeting decided to hold the Spring Meeting in Ann Arbor Michigan. Due some technical difficulties the meeting was moved to Maryland.

At the end of March 1979 many Circus members met and presented their results at the Austin at the International Ticom Conference. This caused that the Circus Spring meeting had less lectures and more time for them. The discussion during the presentation was very live so that it almost came branches for the lecturers.

Today evening we all enjoyed  
a good dinner in Golden Bull  
Restaurant and then a few drinks  
in Kellogg's home

R Fall, being in Taos, did send  
his poem dedicated to the Spry  
meeting. It is presented below

There once was a fellow named Strom  
who used rushes not grass to form  
With considerable ingenuity  
he got approximations converging  
but only in some napkin corners

Pres  
James Douglas Jr.  
Melvin Ciment  
Markell Luskin  
Douglas N. Arnold  
Ragnar Winther

Henrik Borch  
Joe Pasciak  
Michael Vogelius  
B. Kellogg

Mary Fath Wheeler  
Richard Cevany  
Graham Carey  
Stephen Leventhal

Alan E. Berger

Art. H. Lay

Jim Hale on 4-th order alternating direction  
Dianne O'Leary approximation for the heat equation

Ridgway Scott  
Alfred Schatz

Abdul K. Aziz

Brooke Stephens

Dick Morgan

William Symmeyals

Talks in Tech

Jim Douglas Experiments with Galerkin methods for 1-D Water Waves

Douglas-N Arnold An Interior Penalty Finite Element Method with Discontinuous

Ragnar Winther A Stable finite element method for  
symmetric hyperbolic equations.

Richard Ewing Alternating-direction Galerkin Methods  
for Quasilinear Parabolic Problems (with Jim Bramble)

Alan Berger Tridiagonal schemes for a singular  
perturbation problem; Numerical experiments and  
some comments on an "upwind" FEM of Hughes  
(with Jay M. Solomon, Melvin Ciment, Stephen H. Leventhal & Bernard Wender)

Arthur G. Werschulz Improved Convergence of Finite-Element Methods  
for the Discrete-Ordinate Neutron Transport Equation  
(with A. K. Aziz)

Joe Pasciak - On 4'th order alternating direction  
type approximation for the heat equation

Michael Vogelius - on dimensionally reduced models  
for P.D.E's

B. Kellogg - least squares for 1DIV systems  
(with Aziz - Stephens)

I. Babuska - Lower Updr and Jacobian  
a-posteriori error estimates  
for a 1-D model problem

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Finite Element Circus Meeting  
Cornell University  
November 16-17, 1979

The meeting was held at Clark Hall on the Cornell campus. Thirty circus members attended and twenty-two presented talks. After six talks on Friday morning a buffet lunch was held at the North Campus Union, compliments of the Cornell group. In the afternoon seven additional talks were presented. Following these talks, Jim and Peggy Bramble were the gracious hosts of a cocktail party at their home. The group then moved to the Old Port Harbor restaurant for an excellent dinner.

The group convened again on Saturday at 9 A.M. It was decided to hold the Spring 1980

meeting at the Univ. of Michigan  
with Mitch Lushkin and Ridge Scott  
the hosts. The Fall 1980 meeting  
will then be held at Rutgers Univ.  
with Rick Falk the host.

In keeping with a recently  
established circus tradition, Bruce  
Kellogg contributed the following poem.

When far above Cayuga's waters,  
The travelling circus convened,  
Errors were cowed,  
By aitches proud,  
and spaces reigned supreme.

After 9 additional talks,  
the meeting closed.

*Signatures*

Dan B. Wahlbeck + Jacques Belair  
for Calphac Solutions +  
*Pulse* + Stephen Leventhal +  
Q. K. Qazi Alan Berger +  
Jim Bramble John Adams +  
Rick Falk + Thomas Russell +  
Steve Serbin + Peter Sammons +  
A. Schatz + John Bell  
B. Kellogg Larry Bales  
Mitchell Luskin + Charlie Van Loan  
Dick Ewing Stephen Hiller  
Diana & Skaggs + Oliver A. McGraw +  
Todd Dupont  
Mary Janett Wheeler +  
Douglas N Arnold +  
John Douglas Jr. +  
Rolf Rannacher +  
Michael Vogelius +  
Tony MILLER +  
Mark Friedman +  
FERRY de Jong

Quasi-Optimal Estimates in  $L_\infty$   
for Galerkin Solutions (joint with A.-H. Schatz)

Error Estimates For the Numerical  
Identification of a Variable Coefficient  
(R. Falk)

"Almost" Closing the Book on the Cosine Methods

Singular perturbations ( $L_p$  estimates) (joint with L. Wahlbin)

An Adaptive Time Discretization

Strategy for Parabolic Problems  
(talk by M. Luskin, joint work with  
K. Aziz and I. Babuška)

C. Hayes -

numerical Singularity resulting from  
reduced integration techniques

Todd Dupont - Mesh modification for Galerkin methods

Mary F. Wheeler - Mixed Methods for Incompressible  
Displacement Problems in Porous Media

(joint with Jim Douglas, Jr. and Richard Ewing)

Doug Arnold - Adaptive time-step selection for Sobolev equations

Jim Douglas Jr. - combining the method of characteristics  
with finite differences for transport dominated diffusion problems

Rolf Rannacher: Some results for finite element approximation of nonlinear problems.

Michael Vogelius: On methods of dimensional reduction.

Tony MILLER : A-posteriori error estimates.

Mark Friedman: Some results on integral nonlinear magnetostatic equation.

Stephen Leventhal: An Exponential Operator Compact Implicit Method

Alan Berger Resolution of the Boundary Layer in a Singular Perturbation Problem (joint work with Jay Solomon and Melvyn Ciment)

John Osborn Eigenvalue approximation by mixed and hybrid methods

Tom Russell Efficient time-stepping methods for miscible displacement problems in porous media  
(joint with Richard Ewing)

Peter Sammon Report on Efficient Single Step + Multistep Schemes for Nonlinear Parabolic Equations  
(Joint work with J. Bramble).

Oliver McBryan Fingered Solutions for immiscible flow in porous media.

Ivo Babuska A-posteriori estimates for elliptic PDE