

Question:

1. Using a diagram of the unit circle and the Pythagorean theorem, show that

$$\sin^2(\theta) \leq (1 - \cos(\theta))^2 + \sin^2(\theta) \leq \theta^2$$

2. Use part 1) to show that:

$$\sin^2(\theta) \leq 2(1 - \cos(\theta)) \leq \theta^2$$

3. Use part 2) to prove that:

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos(\theta)}{\theta} = 0$$

and

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos(\theta)}{\theta^2} = \frac{1}{2}$$

Question:

Suppose $f(x) = x^3 + x - 1$.

- a) Explain why f has a root in the interval $[0, 1]$.
- b) Suppose A is a constant and $g(x) = x^3 + x - 1 + Ax(x - 1)(2x - 1)$. Show that g has at least one root in the interval $[0, 1]$.
- c) Calculate $g(\frac{1}{3})$ and $g(\frac{2}{3})$. If A is large enough, show that g must have three roots in the interval $[0, 1]$.