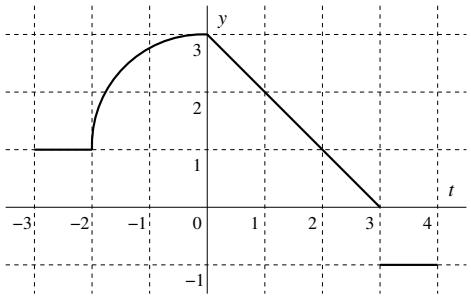
Workshop Problems–November 30

1. Let $F(x) = \int_0^x e^{(t^2)} dt$.

   (a) Compute $\lim_{x\to\infty} \dfrac{xF(x)}{e^{(x^2)}}$.

   (b) Compute $\lim_{x\to 0} \dfrac{F(x)}{xe^{(x^2)}}$.

2. Below is the graph of a function f whose domain is $[-3, 4]$. The graph is made of straight line segments, except for that part of the graph between -2 and 0 which is a quarter circle centered at $(0, 1)$.



   Suppose F is defined by $F(x) = \displaystyle\int_0^x f(t)\, dt$. Sketch the graph of F as well as possible. What are the x-intercepts of F? Determine the intervals on which F is

   (a) continuous

   (b) differentiable

   (c) increasing/decreasing

   (d) concave up/down

   Relate all these answers to the graph of f.

3. A ball moves in a vertical air tunnel in response to air flow in the tunnel. Suppose that $y(t)$ represents the vertical position of the center of the ball above 'ground level'. (Note: It is possible for the ball to be below ground level.) Air is fed into the tunnel at varying rates beginning at $t = -2$ and ending at $t = 14$. Assume that it is known that $y(0) = 10$ and that the vertical velocity of the center of the ball is given by

$$v(t) = \begin{cases} t^2 - t - 6 & \text{if} \quad 0 \le t \le 4 \\ (26 - 2t)/3 & \text{if} \quad 4 \le t \le 13 \end{cases}$$

   (a) Find $y(2)$ and $y(8)$.

   (b) From $t = 0$ to $t = 13$, at what instant of time does the ball reach the lowest point in its trajectory? At what instant does the ball reach the highest point in its trajectory?