Workshop Problems–October 19

- 1. An airplane is flying straight and level at an altitude of 6 miles. At noon it passes directly over a radar station. Some time later the radar operator sees that the (straight-line) distance between the plane and the station is 10 miles and is increasing at 240 miles per hour. What is the speed of the plane at that moment?
- 2. Two circles as shown in the picture below have the same center. The inner circle has radius r which is increasing at the rate of 3 inches per second. The outer circle has radius R which is increasing at the rate of 2 inches per second. Suppose that A is the area of the region *between* the circles. At a certain time t_0 , the smaller radius r is 7 inches and the larger radius R is 10 inches. What is A at this time? How fast is A changing at time t_0 ? Is it increasing or decreasing?



3. We have two ant worlds A and B. They are both $(0, \infty)$ as sets. Ant Enna is from world A and Ant Artica is from world B. Secretly these two worlds are the 'same', but we don't know this at the beginning of the universe. Alien Captain JaBau with his keen observation skills figures out that these two worlds are indeed the 'same', and are related as follows. Suppose a is a number in $(0, \infty)$ that represents a point in world A. Then the point it represents in world B is

$$b = f(a) = \tan\left(\frac{\pi a}{2(a+1)}\right) + ae^a.$$

Now solve the following problems.

- (a) Ant Enna is at a = 5 in world A. Where is her mirror image in world B? Ant Artica is at b = 1 + e in world B. Where is his mirror image in world A?
- (b) Find f'(a) and argue that f'(a) is positive for any a > 0.
- (c) Ant Artica does his morning jog at a constant speed of 1 unit per minute in world B. Find out how fast his mirror image is jogging in world A, when Ant Artica is at b = 1 + e in world B.
- (d) Why are these two worlds the 'same'? What can we say about the function f? How does the mirror image of ant Enna change as she moves on $(0, \infty)$?