

Workshop Problems—September 21

1. Let $f(x) = \frac{\sqrt{2 - \sqrt{4 - x^2}}}{x}$.
 - (a) Find $\lim_{x \rightarrow 0^+} f(x)$ and $\lim_{x \rightarrow 0^-} f(x)$.
 - (b) Sketch the graph of $y = f(x)$ in the viewing window $[-2, 2] \times [-1, 1]$.
 - (c) Use the graph to check your answer to (a). Explain any interesting behavior, particularly involving signs.
2. Find the limit in each of the following cases.
 - (a) $\lim_{x \rightarrow 0^+} \sqrt{x} e^{\cos(\pi/x)}$. (Hint: Use the Squeeze Theorem.)
 - (b) $\lim_{x \rightarrow 0} \frac{4 \sin x - \sin^2 x}{2x}$.
 - (c) $\lim_{x \rightarrow 0} \frac{\sin(4x) \sin(7x)}{2x \sin(3x)}$.
3. Let $S(x) = x^2$.
 - (a) Consider the function T defined by

$$T(x) = \begin{cases} S(x) & \text{if } x \neq 3, \\ 7 & \text{if } x = 3. \end{cases}$$

- Sketch the graph of T . What is $\lim_{x \rightarrow 5} T(x)$? What is $\lim_{x \rightarrow 3} T(x)$? Justify your answers.
- (b) An evil interstellar visitor changes one million values of S and creates a new function V . What can be said about $\lim_{x \rightarrow a} V(x)$ for all values of a ? Justify your answer.
4. (a) Using a diagram of the unit circle and the Pythagorean theorem, show that

$$\sin^2(\theta) \leq (1 - \cos(\theta))^2 + \sin^2(\theta) \leq \theta^2$$

- (b) Use part a) to show that:

$$\sin^2(\theta) \leq 2(1 - \cos(\theta)) \leq \theta^2$$

- (c) Use part b) to prove that:

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos(\theta)}{\theta} = 0$$

and

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos(\theta)}{\theta^2} = \frac{1}{2}$$