## Math 151, Quiz # 9 Solutions, November 5, 2013

1. Find all local minima and maxima for the function  $g(x) = \frac{x+1}{x^2+4}$  which is defined on the whole real line. Solution: We use the quotient rule to compute  $g'(x) = \frac{x^2+4-(x+1)\cdot 2x}{(x^2+4)^2} = \frac{-x^2-2x+4}{(x^2+4)^2}$ . If we want to find x such that g'(x) = 0, it suffices to set the numerator to zero (note that the denominator is never zero). Then we have  $-x^2 - 2x + 4 = 0$ , or equivalently,  $x^2 + 2x - 4 = 0$ . Using the quadratic equation we find the roots:  $\frac{-2\pm\sqrt{4+16}}{2} = -1 \pm \sqrt{5}$ . Note that one of these is negative and one positive. To determine the behavior at these points we test intermediate points. Note that  $-10 < -1 - \sqrt{5} < 0 < 1 + \sqrt{5} < 10$ . So if determine the sign of g'(x) at each of -10, 0 and 10 we can determine the behavior at our critical points. Note that when one plugs each of these into g' it suffices to plug them into the numerator since the denominator is always positive. One can check that g'(10) > 0, g'(0) > 0 and g'(10) < 0. Thus around the first critical point,  $-1 - \sqrt{5}$  the function g is increasing to the left and decreasing to the right. So  $-1 - \sqrt{5}$  is a local minimum. Around the second critical point,  $1 + \sqrt{5}$  the function is increasing then decreasing so this is a local maximum.

**2.** Evaluate the following limits.

**a)**  $\lim_{x\to\infty} \frac{e^x}{x+5}$ . Solution: Note that the numerator and denominator are approaching  $\infty$ . So we can apply L'Hôpital's rule to find the equivalent limit  $\lim_{x\to\infty} \frac{e^x}{1} = \infty$ .

**b)**  $\lim_{x \to 1} \frac{e^x - e}{\ln(x)}$ . Solution: Note that the numerator and denominator are approaching 0 as  $x \to 1$ . So we can apply L'Hôpital's rule to find the equivalent limit  $\lim_{x \to 1} \frac{e^x}{1/x} = \frac{e}{1} = e$ .

c)  $\lim_{x \to \pi} \sin(x) \csc(x)$ . Solution: Note that, where defined,  $\csc(x) = \frac{1}{\sin(x)}$ . So this equals  $\lim_{x \to \pi} 1 = 1$ .