Math 151, Quiz # 7, October 22, 2013

1. Let $y = \cos(\pi x + \ln(x))$. Find $\frac{dy}{dx}$. **Solution:** $\frac{dy}{dx} = -\sin(\pi x + \ln(x))\left[\pi + \frac{1}{x}\right]$.

2. Let $x^2 e^y = x\cos(y)$. Find $\frac{dy}{dx}$. **Solution:** Differentiating both sides with respect to x we find

$$2xe^y + x^2 e^y \frac{dy}{dx} = \cos(y) - x\sin(y)\frac{dy}{dx}.$$

Rearranging terms:

$$x^2 e^y \frac{dy}{dx} + x\sin(y)\frac{dy}{dx} = \cos(y) - 2xe^y.$$

We can factor out $\frac{dy}{dx}$ from the left hand side and then divide to obtain:

$$\frac{dy}{dx} = \frac{\cos(y) - 2xe^y}{x^2 e^y + x\sin(y)}.$$

3. A right triangle is growing. The legs have lengths x and y (which change with time t). If at a certain time, $x = 5$, $y = 6$, $\frac{dx}{dt} = 3$ and $\frac{dy}{dt} = 10$ find the rate at which the area of the triangle is increasing at this time. **Solution:** Let $A = \frac{1}{2}xy$ be the area of the triangle. Differentiating we find $\frac{dA}{dt} = \frac{1}{2}\left(\frac{dx}{dt}y + \frac{dy}{dt}x\right)$. Now we can enter our given values to find $\frac{dA}{dt}$ at the given time

$$\frac{dA}{dt} = \frac{1}{2}(3 \cdot 6 + 10 \cdot 5) = 34.$$