Math 151, Quiz # 6, October 15, 2013

1. Let $y = \ln(\tan(x))$. Find $\dfrac{dy}{dx}$. **Solution:** $\dfrac{dy}{dx} = \dfrac{1}{\tan(x)}\sec^2(x) = \dfrac{\cos(x)}{\sin(x)} \cdot \dfrac{1}{\cos^2(x)} = \dfrac{1}{\cos(x)\sin(x)}$.

2. Let $f(x) = x^4 + 2x^3 + 2x^2 + x + 1$. Let $g(x) = f^{-1}(x)$ (the inverse function). Find $g'(1)$. **Solution:** Note that the problem should have restricted the domian of f to $[-1/2, \infty)$ on which it is increasing and therefore one to one. From the inverse function theorem we have $g'(1) = \dfrac{1}{f'(g(1))}$. Now to find $g(1)$ we need to find x such that $xt + 2x^3 + 2x^2 + x + 1 = 1$. We see that $x = 0$ works. So $g(1) = 0$ since $f(0) = 1$. Now $f'(x) = 4x^3 + 6x^2 + 4x + 1$. So $f'(0) = 1$. Thus $g'(1) = \dfrac{1}{f'(g(1))} = \dfrac{1}{1} = 1$.

3. Find an equation of the tangent line to the curve $x^2e^y + ye^x = 4$ at the point $(2, 0)$. Hint: use implicit differentiation to find $\dfrac{dy}{dx}$. **Solution:** Differentiating both sides with respect to x and then simplifying we find

$$\begin{aligned}
2xe^y + x^2e^y\frac{dy}{dx} + \frac{dy}{dx}e^x + ye^x = 0 \\
x^2e^y\frac{dy}{dx} + e^x\frac{dy}{dx} &= -2xe^y - ye^x \\
(x^2e^y + e^x)\frac{dy}{dx} &= -2xe^y - ye^x \\
\frac{dy}{dx} &= \frac{-2xe^y - ye^x}{x^2e^y + e^x}.
\end{aligned}$$

So at $(2, 0)$ the slope is $\dfrac{-2 \cdot 2 \cdot e^0 - 0 \cdot e^2}{2^2e^0 + e^2} = \dfrac{-4}{4 + e^2}$. Thus using point-slope form, the line we are seeking is

$$y = \frac{-4}{4 + e^2}(x - 2).$$