Math 151, Quiz # 5 Solutions, October 1, 2013

1. Evaluate the following limits:

a) $\lim\limits_{x\to\infty} \dfrac{5x^3 - 6x^2 + 1}{4x^3 - 2x + 2} = \lim\limits_{x\to\infty} \dfrac{x^3(5 - 6/x + 1/x^3)}{x^3(4 - 2/x^2 + 2/x^3} = \lim\limits_{x\to\infty} \dfrac{5x^3}{4x^3} = \dfrac{5}{4}$.

b) $\lim\limits_{x\to\infty} \dfrac{x^3 - x^2 + 7}{\sqrt{x^4 - 3}} = \lim\limits_{x\to\infty} \dfrac{x^3(1 - 1/x + 7/x^3)}{\sqrt{x^4(1 - 3/x^4)}} = \lim\limits_{x\to\infty} \dfrac{x^3}{\sqrt{x^4}} = \lim\limits_{x\to\infty} x = \infty$. Note that the highest power in the numerator is 3 and in the denominator is $\sqrt{4} = 2$. So it makes sense that the numerator dominates and the limit is infinity.

c) $\lim\limits_{x\to\infty} \dfrac{17x^6 - x^4 + x^2 + x + 1}{x^9 - x - 5} = 0$. The power in the denominator is 9 whereas the power in the numerator is 6. Like above you can show this by factoring out the highest power in the numerator and denominator. If you do this you will end up with $\lim\limits_{x\to\infty} \dfrac{17}{x^3} = 0$.

2. Find the derivative of $(x + 1)(x^2 + 3x - 5)$. If you use the product rule you obtain

$$1 \cdot (x^2 + 3x - 5) + (x + 1)(2x + 3) = x^2 + 3x - 5 + 2x^2 + 5x + 3 = \mathbf{3x^2} + \mathbf{8x} - \mathbf{2}.$$

Alternatively, you can multiply out the expression:

$$(x + 1)(x^2 + 3x - 5) = x^3 + 4x^2 - 2x - 5$$

and then differentiate thsi to find the answer in bold above.