Math 151, Quiz # 2, September 10, 2013

1. Find the equation of the line that passes through the point (3, -4) and is parallel to the line y = 3x - 6.

Solution: The slope of the given line is 3, so a parallel line will have slope 3. We can use point slope formula y - (-4) = 3(x - 3). This simplifies to y = 3x - 13.

2. Find all values of x on the interval $[0, 2\pi]$ such that $\tan(x) = \sqrt{3}$.

Solution: In the first quadrant, $\tan(x) = \sqrt{3}$ when $x = \pi/3$. We can see this from the $\pi/3$ - $\pi/6$ - $\pi/2$ special right triangle. Recall that tan is positive in the first and third quadrant. So $\pi + \pi/3 = 4\pi/3$ also is a solution. So the solution set is $\{\pi/3, 4\pi/3\}$.

3. Let f(x) = ax + 3 for some constant a. For what values of a does this function have an inverse? Compute $f^{-1}(x)$ when a = 2. What is $f^{-1}(19)$? Use the function f(x) to double check your answer.

Solution: We can solve for x as $x = \frac{f(x)-3}{a}$. This is well defined when $a \neq 0$. So the function is invertible when $a \neq 0$. Another way to see this is to note that f(x) is the equation of a line and thus passes the horizontal line test, except if it is itself a horizontal line. So as long as the slope is not zero, f(x) is invertible.

We computed above, that $f^{-1}(x) = \frac{x-3}{a}$. So if a = 2, we have $f^{-1}(x) = \frac{x-3}{2}$. So $f^{-1}(19) = \frac{19-3}{2} = \frac{16}{2} = 8$. We can check this by computing $f(8) = 2 \times 8 + 3 = 19$.