Math 151, Quiz # 10, November 19, 2013

1. Evaluate the following sums. Show your work.

a) $\displaystyle\sum_{j=0}^{5} j^2$. **Solution:** $\displaystyle\sum_{j=0}^{5} j^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 1 + 4 + 9 + 16 + 25 = 55$.

b) $\displaystyle\sum_{k=0}^{20} (-1)^k$. **Solution:** $\displaystyle\sum_{k=0}^{20} (-1)^k = 1 - 1 + 1 - 1 + \ldots + 1$. There are 11 ones in the sum (corresponding to even exponents) and 10 negative ones in the sum (corresponding to odd exponents). So the sum is $11 - 10 = 1$.

c) Let $f(x) = \displaystyle\sum_{k=0}^{8} x^k$. Write $f'(x)$ in sum notation. **Solution:** The derivative with respect to x of x^k is kx^{k-1}. This is the power rule. The derivative of a sum is the sum of the derivatives. So we have $f'(x) = \displaystyle\sum_{k=0}^{8} kx^{k-1}$. The term at $k = 0$ is 0 (this corresponds to differentiating the constant term of $f(x)$). Thus we can drop this term and write $f'(x) = \displaystyle\sum_{k=1}^{8} kx^{k-1}$.

2. Using a geometrical argument, compute $\displaystyle\int_{0}^{8} (x - 2)\, dx$. **Solution:** The line $y = x - 2$ has y-intercept of -2 and intersects the x-axis at $x = 2$. So from 0 to 8 the region between $x - 2$ and the x-axis forms two right triangles, one below the x-axis and the other above. The first triangle has legs of lengths 2 and 2 so has signed area -2. The second has legs of length 6 and 6 so has signed area 18. Thus the integral evaluates to $18 - 2 = 16$.