Review Problems for Midterm 2

Please note that this set of problems does *not* necessarily cover all topics that may appear on your exam.

1. Let $f(x) = x^3 + 3x + 2$. Let $g(x) = f^{-1}(x)$ be the inverse function. Find $g'(2)$ and $g'(6)$.
2. Let $y = e^{x+\sin(x^2)}$. Find $\frac{dy}{dx}$.
3. Let $f(x) = \ln(3 - 5x^2)^3$. Find $f'(x)$.
4. Let $g(x) = \sin^2(\sqrt{x^2+1}) + \cos^2(\sqrt{x^2+1})$. Find $g'(x)$.
5. Let $e^x y + y^2 x + x^3 \cos(y) = 0$. Find $\frac{dy}{dx}$. Show that $(0, 0)$ is on the curve and that there is a vertical tangent at this point.
6. Consider the circle $(x-2)^2 + (y-4)^2 = 25$. Where on this circle is the slope of the tangent line equal to 1?
7. Let $h(x) = \cos(x) + \sin(x)$. Find the minimum and maximum values attained by $h(x)$ on the interval $[0, 2\pi]$.
8. Let $y = xe^{-x}$ be defined on $[0, 2]$. Find the min and max of this function.
9. Let $f(x) = x^2 + 2x + 3$. Find the average rate of change of this function on the interval $[1, 3]$. The mean value theorem says there is some point c on this interval at which $f'(c)$ attains this average rate of change. Find such a value c.
10. Let $f(x) = \frac{1}{4}x^4 + 2x^3 - 3x^2 + 3x - 1$. Find all inflection points of $f(x)$.
11. Two runners start running at the origin. One runs due North at 8 m/s. The second runs due East at 6m/s. How fast are they moving apart from each other when the first runner is 80 meters from the origin and the second is 60 meters from the origin?
12. An object moves along the curve $y = e^x$. At what point(s) is the object moving twice as fast in the y direction as it is in the x direction?
13. Let $g(x) = x\cos(x) + e^x + 3$. Using a linear approximation, estimate $g(0.05)$.
14. Let $h(x) = x^3 - 2x^2 + 3x - 4$. We want to find its roots using Newton's Method. Write down a recursive formula for the value of x_{n+1} in terms of x_n. If $x_0 = 1$, find the value of x_1?
15. Let $f(x) = \sin^2(x)$. Find the critical points, local maxima and minima, global maximum and minimum and inflection points on the interval $[0, 2\pi]$.
16. Evaluate $\lim_{x\to\infty} \dfrac{x^2\ln(x) + x}{e^x}$.
17. Evaluate $\lim_{x\to 0} \dfrac{\sin(x)\cos(x)}{e^x - 1}$.
18. Evaluate $\lim_{x\to\infty} \dfrac{x^{100}}{e^x}$.
19. Evaluate $\lim_{x\to 0} x^2\ln(1/x^2)$.