Solutions to Review Problems for Math 151 Final

1. A rocket ship blasts off from earth with acceleration function given by $a(t) = 3t^2 + 4t + 2$. Assume that it launches from a height of 0 meters and is initially at rest. Find the height of the rocket at time $t = 10$ seconds. **Solution:** We can integrate $a(t)$ to find the velocity function $v(t) = \int a(t)\ dt = t^3 + 2t^2 + 2t + C$. To find C note that we are told that the rocket ship is initially at rest, so $v(0) = 0$. Thus, $C = 0$. Next we can integrate the velocity function to find the height function $h(t) = \int (t^3 + 2t^2 + 2t)\ dt = \frac{1}{4}t^4 + \frac{2}{3}t^3 + t^2 + D$. We are told $h(0) = 0$ so $D = 0$. Thus $h(t) = \frac{1}{4}t^4 + \frac{2}{3}t^3 + t^2$. So $h(10) = \frac{9800}{3}$.

2. Find the area in the plane enclosed by the curves $y = x^2$ and $y = 10 - 3x$. **Solution:** Here is a plot of the two functions: So the region is below the line and above the parabola

Figure 1: $y = 10 - 3x$ is plotted in green, $y = x^2$ is plotted in red.

between the two intersection points. To find these points we set the curves equal to each other: $10 - 3x = x^2 \Rightarrow x^2 + 3x - 10 = 0 \Rightarrow (x + 5)(x - 2) = 0$ so $x = -5$ and $x = 2$ are the intersection points. So the area is given by $\int_{-5}^{2} \left[10 - 3x - x^2\right]\ dx = 10x - \frac{3}{2}x^2 - \frac{1}{3}x^3\Big|_{-5}^{2} = \frac{343}{6}$.

3. Compute $\int x^2 \sin(x^3)\ dx$. **Solution:** Let $u = x^3$. Then $du = 3x^2\ dx$. Substituting our integral becomes $\int x^2 \sin(u) \frac{du}{3x^2} = \frac{1}{3} \int \sin(u) = -\frac{1}{3}\cos(u) + C = -\frac{1}{3}\cos(x^3) + C$.

4. Let $h(x) = \int_x^{2x} e^{t^2}\ dt$. Find $h'(x)$. **Solution:** Let $g(t)$ be an antiderivative of e^{t^2}. Then $g'(t) = e^{t^2}$. The fundamental theorem of calculus tells us that $h(x) = g(2x) - g(x)$. Differentiating, we have $h'(x) = g'(2x) \cdot 2 - g'(x) = e^{(2x)^2} \cdot 2 - e^{x^2}$.

5. Compute $\int (x-2)\sqrt{x+4}\,dx$. **Solution:** Let $u = x+4$. Then $du = dx$. When we make the substitution the radical will become $\sqrt{u}$ and for the other term we can substitute $x = u-4$. Thus, our integral becomes $\int (u-4-2)\sqrt{u}\,du = \int (u-6)u^{1/2}\,du = \int \left(u^{3/2} - 6u^{1/2}\right) du = \frac{2}{5}u^{5/2} - 4u^{3/2} + C = \frac{2}{5}(x+4)^{5/2} - 4(x+4)^{3/2} + C.$

6. Find the area enclosed by the curves $y = x^3$, $y = 8$ and $x = 0$. **Solution:** See the plot below: The curve $y = x^3$ and $y = 8$ intersect at when $x^3 = 2$, so when $x = 2$. Thus we integrate

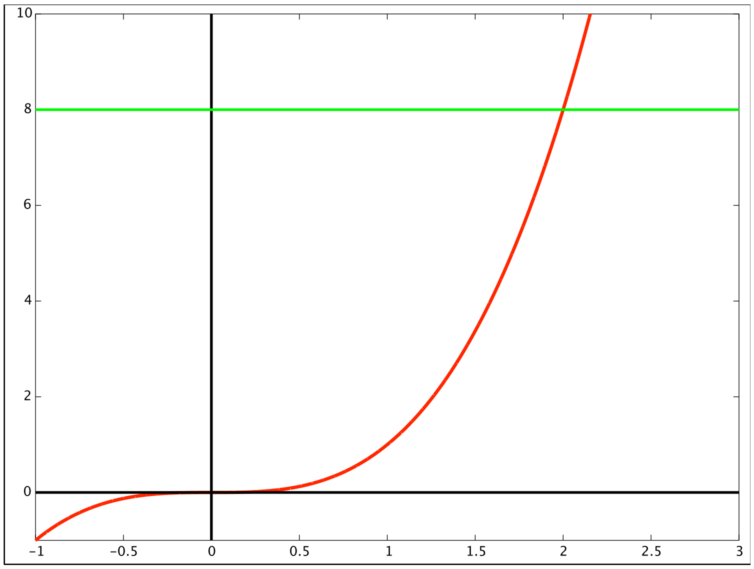

Figure 2: $y = x^3$ is plotted in red, $y = 8$ is plotted in red. The y-axis (the line $x = 0$) is the third curve enclosing the region.

$$\int_0^2 8 - x^3 = 8x - \frac{1}{4}x^4 \Big|_0^2 = 12.$$

7. Write out, but don't compute $R_C(12)$, the Riemann approximation for $\int_0^6 (x^2+1)\,dx$ using 12 rectangles using the center point of the intervals. **Solution:** We have $\Delta x = \frac{6-0}{12} = \frac{1}{2} = 0.5$. Our first interval is $[0, 0.5]$, our second is $[0.5, 1]$, etc. until our last interval is $[5.5, 6]$. The midpoints of our intervals are then $0, 25, 0.75, 1.25, \ldots, 5.75$. So

$$R_C(12) = 0.5\left[(0.25^2+1) + (0.75^2+1) + (1.25^2+1) + \ldots + (5.75^2+1)\right].$$

8. [Note: this topic is not on your final exam] You invest 100 in a bank account that pays 3% interest. If it is compounded continuously compute how long it will take for your money to triple. An expression for the time is fine, you do not need an exact answer. **Solution:** We use the equation $P = P_0 e^{rt}$. We have $P_0 = 100$ and $r = 0.03$. We want to find t such that $P = 300$. So $300 = 100e^{0.03t}$, thus $3 = e^{0.03t}$. Taking the log of both sides: $\ln(3) = 0.03t$ so $t = \frac{\ln(3)}{t}$. Note that the time it takes the money to triple does *not* depend on the initial amount.