151 Final Exam Practice Problems

Problem 1

Suppose from $t = 0$ to $t = \pi$ a particle moves along the x-axis according to the velocity function $v(t) = \sin(t) - \cos(t)$. The particle starts at the origin.

1. Give a function $x(t)$ that describes the particles position over time.
2. What is the furthest the particle travels from the origin?
3. What is the total distance travelled by the particle?

Problem 2

Suppose $F(x) = \int_{-1}^{x^2+3x} \cos(t^2)\ dt$. Find $F'(0)$.

Problem 3

Let $f(x)$ be given by $-2x + 4$ when $x < 2$ and by the upper half of a circle of radius 4 centered at $(6, 0)$ for $2 \leq x \leq 10$.

1. Compute $\int_0^6 f(x)\ dx$.
2. Compute R_6, i.e. the right Riemann sum using six rectanges that estimates the integral given above.

Problem 4

Compute $\int x\sqrt{x+1}\ dx$

Problem 5

Let $g(x) = \frac{e^x - x - 1}{x^2}$ when $x \neq 0$. What value should we define $f(0)$ for f to be continuous.

Problem 6

A right triangle with a vertex fixed at the origin is growing. One leg is growing to the right along the x-axis at a rate of 8 m/s. Another leg is shrinking along the y-axis (moving from positive toward the origin) at a rate of 5 m/s. How fast is the area of the triangle changing when $x = 12$ and $y = 5$? Is it increasing or decreasing? How fast is the hypotenuse changing? Is it increasing or decreasing?

Problem 7

A long river runs North to South. You have 100 feet of fencing with which to make a rectangular enclosure on the East bank. The borders of this enclosure will be the river to the West and three lines of fencing. What is the maximal area that can be formed?

Problem 8

Find:

1. $\int_0^{2\pi} \cos^2(x) + \sin^2(x)\ dx$

2. $\int_0^{2\pi} \cos^2(x) - \sin^2(x)\ dx$

3. $\int x^2(\cos(x^3) - x)\ dx$

Problem 9

Consider the curve $x^2 + y^2 = 3xy - 1$. Show that the point $(1, 2)$ lies on the curve. Find the tangent line at this point.

Problem 10

Consider the equation $x = \cos(x)$. Show that this has a root. Next, if $x_0 = 0$ is an initial guess of the root what would be the next estimate, x_1, using Newton's method.

Problem 11

Use linear approximation to estimate $\sqrt[5]{33}$. Is this an underestimate or an overestimate? Explain.

Problem 12

1. $\lim_{x \to \pi/2} \dfrac{x - \pi/2}{\sin(x)}$

2. $\lim_{x \to 1} \dfrac{\ln(x)}{x - 1}$

3. $\lim_{x \to 0} x^{\sin(x)}$