640:250 Introductory Linear Algebra Midterm Exam #2 April 8, 2015

Name (PRINT):____

ID # (last 4 digits):______________

Signature: ____

Instructions: This is a closed book exam. Show your answers and arguments for your answers in the space provided. You may use the back of the pages also, but indicate clearly any such material that you want graded. No calculators, cell phones, or any other electronic devices may be used during the exam.

Have your photo ID card available for checking. Do not start the exam until instructed to do so.

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Problem	# Points	Score
1	15	
2	25	
3	15	
4	25	
5	20	
Extra Credit	10	
total	100+10	

Problem 1 (15 points total)

Let $A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & -4 \\ 0 & 10 & -6 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 9 \\ -15 \\ -22 \end{bmatrix}$.

a. (8 points) Compute the LU-decomposition of A.

b. (7 points) Use the LU-decomposition found in part (a) to solve $A\vec{x} = \vec{b}$.

Problem 2 (25 points total)

a. (5 points) Let $M = \begin{bmatrix} 3 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & 5 & 4 \end{bmatrix}$. Find $\det M$ by employing cofactor expansion along the first row.

b. (5 points) Let $B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 5 & 5 & -1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$. Find $\det B$. Indicate clearly which row or column you are cofactor expanding along.

c. (5 points) Let $C = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 5 & 4 & 3 & 2 & 1 \\ 6 & 6 & 7 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$. Find $\det C$. Hint: you do not need to employ cofactor expansion. Explain your answer.

d. (10 points total, each part worth 2 points) Let A be a 5×5 matrix with $\det A = 10$. Compute the following:

i. Find $\det(A^2)$.

ii. Find $\det(A^{-1})$.

iii. Let B be the matrix formed by swapping rows 2 and 4 of A. What is $\det B$?

iv. Find $\det(A + A)$.

v. Let D be the matrix formed by multiplying the i-th row of A by i. So the first row is scaled by 1, the second row is scaled by 2, etc. Find $\det(D)$.

Problem 3 (15 points total)

a. (7 points) Let A and B be $n \times n$ matrices that are *not* invertible. Prove that AB is not inveritible.

b. (8 points) A square matrix, M, is said to be *nilpotent* if some power of the matrix equals the zero matrix. That is, there is some positive integer k such that $M^k = 0$. Let M be a nilpotent matrix. Explain why $\det M = 0$.

Problem 4 (25 points total)

Let $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 2 & 4 & 7 \\ 1 & 4 & 5 & 6 \end{bmatrix}$. Parts (a), (b) and (c) refer to this matrix.

a. (5 points) Find a basis for Col$A$. What is dim(Col$A$)?

b. (5 points) Find a basis for Row$A$. What is dim(Row$A$)?

c. (5 points) Find a basis for Null(A)? What is dim(Null$A$)?

d. (10 points) Let B be an invertible, 5×5 matrix. Describe the following:

i. Col(A)

ii. Row(A)

iii. Null(A)

Problem 5 (20 points total)

Let $M = \begin{bmatrix} -7 & 18 \\ -3 & 8 \end{bmatrix}$. Parts (a) and (b) deal with this matrix.

a. (5 points) Find the characteristic polynomial of M and use it to find the eigenvalues of M.

b. (5 points) Find bases for the eigenspaces corresponding to each of the eigenvalues found in part (a).

c. (5 points) Let A be a 5×5 matrix with characteristic polynomial $-(t-5)^3(t^2+8)$. Identify the eigenvalues of A and for each eigenvalue state its algebraic multiplicity. Furthermore, for each eigenvalue λ state the possible values for $\dim(\mathrm{Null}(A - \lambda I_5))$.

d. (5 points) Let B be an $n \times n$ matrix with eigenvalue λ. Show that λ^2 is an eigenvalue of the matrix B^2.

Extra Credit Problem 1 (5 points)

As stated in problem three, a square matrix, M, is said to be *nilpotent* if some power of that matrix equals the zero matrix. That is, there is some positive integer k such that $M^k = 0$. Let M be a nilpotent matrix. Show that all its eigenvalues must be 0.

Extra Credit Problem 2 (5 points)

Prove that for any square matrix A, $\dim(\text{Null}(A)) \leq \dim(\text{Null}(A^2))$. Give an example of a matrix A where $\dim(\text{Null}(A)) < \dim(\text{Null}(A^2))$.