640:250 Introductory Linear Algebra Midterm Exam #1 February 25, 2015

Name (PRINT):______________________________

ID # (last 4 digits):____________

Signature: ______________________________

Instructions: This is a closed book exam. Show your answers and arguments for your answers in the space provided. You may use the back of the pages also, but indicate clearly any such material that you want graded. No calculators, cell phones, or any other electronic devices may be used during the exam.

Have your photo ID card available for checking. Do not start the exam until instructed to do so.

DO NOT WRITE BELOW THIS LINE

Problem	Score
1	
2	
3	
4	
5	
Extra Credit	
total (out of 100)	

Problem 1 (20 points total)

In this problem $A = \begin{bmatrix} 4 & 5 & 2 \\ -1 & 0 & 3 \\ -2 & 1 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 2 & -3 \\ 6 & 7 & -5 \\ -4 & 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 & 1 \\ -7 & 1 & 0 \\ 6 & 2 & 0 \\ 3 & 2 & -3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$.

a. (5 points) For each of $A, B, C, \vec{v}, \vec{w}$ write the size of the matrix. For example, your answer might take the form "Matrix ____ has size ____ by ____."

b. (10 points) Compute each of the following or state that it does not exist (problem continues on next page):

$A + B =$

$C^T =$

$A\vec{w} =$

$A - B^T =$

$B^T\vec{v} =$

$C^T\vec{w} =$

$BA =$

c. (5 points) What is the (2-3) entry of the matrix product AC^T?

Problem 2 (20 points total)

For the following system of equations write the system as a matrix equation $A\vec{x} = \vec{b}$. Indicate clearly the matrix A and the vector $\vec{b}$. Compute the reduced row echelon form of the augmented matrix $[A\ \vec{b}]$. Use this to find the general vector solution.

$$\begin{aligned} 2x_2 - x_3 &= 6 \\ x_1 + 6x_2 - 3x_3 + x_4 &= 20 \\ 4x_1 + 26x_2 - 13x_3 + 5x_4 &= 90 \end{aligned}$$

Problem 3 (20 points total)

A is a 4×5 matrix and $\vec{v} \in \mathbb{R}^5$. In trying to solve the matrix equation $A\vec{x} = \vec{v}$ you form the augmented matrix $\begin{bmatrix} A \ \vec{v} \end{bmatrix}$ and then compute the reduced row echelon form of this matrix and obtain $\begin{bmatrix} 1 & 4 & 0 & 0 & 5 & 7 \\ 0 & 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 0 & 1 & 8 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

a. (4 points) What is rank(A)?

b. (4 points) What is the nullity of A?

c. (4 points) Is $A\vec{x} = \vec{v}$ consistent?

d. (4 points) How many solutions does $A\vec{x} = \vec{0}$ have? Explain.

e. (4 points) Give an example of a matrix in reduced row echelon form that if it were to represent the rref of an augmented system of equations, the system would *not* be consistent.

Problem 4 (20 points total)

For each of the following sets of vectors, determine whether or not they are linearly indepedent. Explain your answer.

a. (7 points) $\left\{ \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix}, \begin{bmatrix} 17 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$

b. (7 points) $\left\{ \begin{bmatrix} -5 \\ 6 \\ 8 \end{bmatrix}, \begin{bmatrix} -11 \\ -12 \\ 20 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ -4 \end{bmatrix} \right\}$. Also: is the span of these vectors equal to $\mathbb{R}^3$? Why/why not.

c. (6 points) $\left\{ \begin{bmatrix} 14 \\ 3.7 \end{bmatrix}, \begin{bmatrix} 2 \\ 6.1 \end{bmatrix} \right\}$. Also determine the span of these vectors.

Problem 5 (20 points total)

a. (5 points) Determine if the matrix $\begin{bmatrix} 12 & -4 \\ 6 & -2 \end{bmatrix}$ is invertible.

b. (10 points) Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & -3 & -3 \\ 0 & -1 & 2 \end{bmatrix}$. Compute A^{-1}. Use A^{-1} to solve the equation $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

c. (5 points) Let A and B be $n \times n$ matrices that are *not* invertible. Prove that AB is not inveritible.

Extra Credit 1 (6 points)

Let $A_1 = [1]$, $A_2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $A_3 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ and in generaly A_n is the $n \times n$ matrix containing the entries $1, 2, 3, \ldots, n^2$ with the first row $[1, 2, \ldots, n]$, the second row $[n+1, n+2, \ldots, 2n]$, etc. Determine for which values of n the matrix A_n is invertible.

Extra Credit 2 (4 points)

Recall that a $n \times n$ matrix A is said to be *diagonal* if $a_{i,j} = 0$ if $i \neq j$. That is each of the non-diagonal entries of A are zero. True or false: If $A^4 = I_5$ (the 5×5 identity matrix) then A must be the identity matrix.