Review Problems for Math 250 Final

1. Let $A = \begin{bmatrix} 5 & -3 \\ 6 & -4 \end{bmatrix}$. Compute A^{100}.

2. Find the distance from the point $(2, 5)$ to the line $y = \frac{1}{3}x$.

3. Find a unit vector perpendicular to both $\vec{v} = \begin{bmatrix} 3 \\ 3 \\ 0 \\ 1 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 2 \end{bmatrix}$.

4. Let $M = \begin{bmatrix} 1.5 & 2.5 \\ -0.5 & 4.5 \end{bmatrix}$. Compute the QR factorization of M. Use it to solve $M\vec{x} = \begin{bmatrix} 2 \\ -22 \end{bmatrix}$.

5. Let $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 7 \end{bmatrix}$. Let $W = \text{Span}\{\vec{v}, \vec{u}\}$. Find a basis for $W^{\perp}$.

6. Let $\{\vec{v}_1, \ldots \vec{v}_5\}$ be a basis for a subspace, V, of $\mathbb{R}^8$. What is $\dim(V^{\perp})$?

7. Consider the plane, $P = \text{Span}\left\{ \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \right\}$. Show that $\vec{v} = \begin{bmatrix} -9 \\ 4 \\ 6 \end{bmatrix}$ is *not* in P.
   Find the closest point in P to $\vec{v}$.

8. Let $x_1, \ldots x_n$ be a list of real numbers. Prove that
$$(x_1 + x_2 + \ldots + x_n)^2 \leq n \left(x_1^2 + x_2^2 + \ldots + x_n^2 \right).$$
   Hint: use the Cauchy-Schwarz Inequality.