Math 151, Quiz #2, September 21, 2017

1. What is $\lim_{x\to\infty} \frac{x^2+3x-5}{4x^2-5x+6}$?

Solution: Factor out an x^2 from numerator and denominator:

$$\frac{x^2+3x-5}{4x^2-5x+6} = \frac{x^2(1+3/x-5/x^2)}{x^2(4-5/x+6/x^2)} = \frac{(1+3/x-5/x^2)}{(4-5/x+6/x^2)}.$$

As $x \to \infty$, terms like $3/x$ and $5/x^2$ tend to 0. Thus

$$\lim_{x\to\infty} \frac{x^2+3x-5}{4x^2-5x+6} = \frac{1+0-0}{4-0+0} = \frac{1}{4}.$$

2. What is $\lim_{x\to 3} \frac{2x^2-5x-3}{5x-15}$?

Solution: We cannot plug in directly since both the numerator and denominator approach zero. So we factor:

$$\lim_{x\to 3} \frac{2x^2-5x-3}{5x-15} = \lim_{x\to 3} \frac{(x-3)(2x+1)}{5(x-3)} = \lim_{x\to 3} \frac{2x+1}{5} = \frac{7}{5}.$$

3. What is $\lim_{x\to 1} \frac{\sin(\log(x))}{e^x}$?

Solution: Here we can just plug in. Recall that $\log(1) = 0$. So

$$\lim_{x\to 1} \frac{\sin(\log(x))}{e^x} = \frac{\sin(\log(1))}{e} = \frac{\sin 0}{e} = \frac{0}{e} = 0.$$

4. Find all values of $0 \le x \le 2\pi$ such that $\sin(2x) = \cos(x)$.

Solution: Recall that $\sin(2x) = 2\sin(x)\cos(x)$. So we want to solve $2\sin(x)\cos(x) = \cos(x)$. This is valid when $\cos(x) = 0$. Otherwise, we can divide both sides by $\cos(x)$ and obtain $2\sin(x) = 1$ and so $\sin(x) = 1/2$. On $[0, 2\pi]$, $\cos(x) = 0$ at $x = \pi/2$ and $3\pi/2$. On $[0, 2\pi]$, $\sin(x) = 1/2$ at $x = \pi/6$ and $5\pi/6$. So our set of solutions is $\{\pi/6, \pi/2, 5\pi/6, 3\pi/2\}$.