Math 135, Quiz #9 Solutions, April 7, 2014

1. Compute $\lim_{x\to\infty} \dfrac{e^x}{x^2+4x+1}$

Solution: As the numerator and denominator approach ∞ we can apply L'Hopital's rule to obtain $\lim_{x\to\infty} \dfrac{e^x}{2x+4}$. The numerator and denominator still approach ∞, so we apply L'Hopital's rule again obtain $\lim_{x\to\infty} \dfrac{e^x}{2} = \infty$.

2. Compute $\lim_{x\to 0} \dfrac{\sin(3x)}{\tan(7x)}$.

Solution: As the numerator and denominator approach 0 we can apply L'Hopital's rule to obtain $\lim_{x\to 0} \dfrac{3\cos(3x)}{7\sec^2(7x)} = \dfrac{3}{7}$.

3. Compute $\lim_{x\to 0} \dfrac{\sin^2(5x)}{x+\cos^3(6x)}$.

Solution: We can plug in directly. We obtain $\dfrac{0}{0+1} = 0$.

4. Two positive numbers sum to 12. What is the largest possible value of the square of the first times the second?

Solution: Call the first number x and the second y. We have that $x+y=12$. We want to maximize $P = x^2y$. We can solve for y in terms of x and obtain $y = 12-x$. Then $P = x^2(12-x) = 12x^2 - x^3$ which is now a function of just x. We want to maximize P. Our domain is $(0,12)$ since x and y are positive. We compute $P' = 24x - 3x^2$ which we set equal to 0. So we have $24x - 3x^2 = 0$ and factoring $3x(8-x) = 0$ and we obtain 0 and 8 as critical numbers. Note that $P(0) = 0$ and in fact 0 is not in our domain. However $P(8) = 8^2 \cdot 4 = 256$ which is our max. We should also check that $P(12) = 0$.