Math 135, Quiz #8 Solutions, March 31, 2014

1. Let $f(x)$ be a function with derivative $f'(x) = x^4(e^x - 5)$. Find all critical numbers of f and determine if they are a relative max, relative min, or neither.

Solution: We set $x^4(e^x - 5) = 0$ and find $x = 0$ or $e^x = 5$ so $x = \ln(5)$. So these are our critical numbers. We note that $e \approx 2.7$ so $1 < \ln(5) < 3$. We will use the first derivative test. Our first interval is $(-\infty, 0)$ We note $f'(-1) = (-1)^4(e^{-1} - 5)$ which is a positive times a negative, thus negative. Therefore f is decreasing on this interval. Our next interval is $(0, \ln(5))$ and 1 is on this interval. So $f'(1) = e - 5 < 0$. So f is decreasing on this interval. Our final interval is $(\ln(5), \infty)$. We note that 3 is on this interval and that $f'(3) = 3^4(e^3 - 5)$. Since $e^3 > 5$, we see that $f'(3)$ is positive. Thus f is increasing on this final interval. So 0 is neither a max nor min, but $\ln(5)$ is a local min.

2. Evaluate the following limits

a) $\displaystyle\lim_{x\to\infty} \frac{6x^3 - x^2 + 17x - 1}{2x^3 + 5x^2 + 5} = \lim_{x\to\infty} \frac{x^3(6 - 1/x + 17/x^1 - 1/x^3)}{x^3(2 + 5/x + 5/x^3)} = \lim_{x\to\infty} \frac{6}{2} = 3.$

b) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x^3 + 9x + 1}}{x^2 - 3x - 1} = \lim_{x\to\infty} \frac{\sqrt{x^3(1 + 9/x^2 + 1/x^3}}{x^2(1 - 3/x - 1/x^2)} = \lim_{x\to\infty} \frac{x^{1.5}}{x^2} = \lim_{x\to\infty} \frac{1}{\sqrt{x}} = 0.$

c) $\displaystyle\lim_{x\to 3^+} \frac{x^3 - 10}{3 - x}$. As $x \to 3^+$ the numerator approaches $3^3 - 10 = 17$. The denominator approaches 0 from the negative side since $3 - x < 0$ if $x > 3$. So the ratio is approaching $-\infty$.