Math 135, Quiz #7 Solutions, March 24, 2014

1. Estimate $\sqrt[3]{65}$.

Solution: Let $f(x) = x^{1/3}$. We know $f(64) = 4$ and want to estimate $f(65)$. We compute $f'(x) = \frac{1}{3}x^{-2/3}$. So $f'(64) = \frac{1}{3} \cdot \frac{1}{16} = \frac{1}{48}$. So the equation of the tangent line at $(64, 4)$ is $y - 4 = \frac{1}{48}(x - 64)$. So we have the linear approximation $L(x) = 4 + \frac{1}{48}(x - 64)$. So $f(65) \approx L(65) = 4 + \frac{1}{48}$. Note that as $\sqrt[3]{64} = 4$ it makes sense that our answer should be a little larger than 4.

2. Let $f(x) = 2x^3 - 9x^2 - 24x + 1$. Find the absolute extrema of f on the interval $[-2, 10]$.

Solution: We compute $f'(x) = 6x^2 - 18x - 24$. We can factor out a 6 and set the derivative equal to zero: $6(x^2 - 3x - 4) = 0$. Factoring the left side we obtain $6(x + 1)(x - 4) = 0$ so $x = -1, 4$ are critical numbers. Thus we will plug in $-1, 4, -2, 10$ into f.

- $f(-2) = -3$
- $f(-1) = 14$
- $f(4) = -111$
- $f(10) = 861$

So the absolute minimum occurs at $(4, -111)$ and the absolute maximum occurs at $(10, 861)$¿