Math 135, Quiz #6 Solutions, March 10, 2014

1. Confirm that the point $(1,0)$ lies on the curve $x^3y + xe^y + y^5 - 1 = 0$. Find the equation of the tangent line at this point.

Solution: To see that $(1,0)$ lies on the curve we plug in $x = 1$ and $y = 0$ to the equation above. We get $0 + 1 + 0 - 1 = 0$ so $(1,0)$ does lie on the curve. Next, we use implicit differentiation to find the derivative. Differentiating we obtain

$$3x^2y + x^3\frac{dy}{dx} + e^y + xe^y\frac{dy}{dx} + 5y^4 = 0.$$

If we solve for $\frac{dy}{dx}$ we obtain

$$\frac{dy}{dx} = \frac{-5y^4 - e^y - 3x^2y}{x^3 + xe^y}.$$

Thus the slope of the tangent line at $(1,0)$ is $\frac{dy}{dx}(1,0) = \frac{0-1-0}{1+1} = \frac{-1}{2}$. So the equation of the tangent line is $y = -\frac{1}{2}(x-1)$.

2. Two runners meet up at a certain point at the start of a run. Alice runs at 8 miles per hour due North and Bob runs at 6 miles per hour due East. At what rate is the distance between Alice and Bob changing half an hour into their run?

Solution: Let $x(t)$ be the distance of Bob from the origin and $y(t)$ the distance of Alice. We have $\frac{dy}{dt} = 8$ and $\frac{dx}{dt} = 6$. Let $R(t)$ be the distnace between Alice and Bob. So $R(0) = 0$. We have by the Pythagorean theorem that $R^2 = x^2 + y^2$. Differentiating, $2R\frac{dR}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$. Before we plug in at $t = 1$ hour we note that $R(1)^2 = x(1)^2 + y(1)^2$. Since Alice is 4 miles North of the starting point and Bob is 3 miles east of the starting point we have $R(1)^2 = 4^2 + 3^2$. Thus $R(1) = 5$. So we have $2 \cdot 5\frac{dR}{dt} = 2 \cdot 3 \cdot 6 + 2 \cdot 4 \cdot 8 = 100$. So $\frac{dR}{dt} = 10$ miles per hour.