Math 135, Quiz # 5 Solutions, Febuary 24, 2014

1. Find the derivative of $g(x) = \sin^2(x) + \cos^2(x) + \sin(x)\cos(x)$. Simplify your answer.

Solution: Recall that $\sin^2(x) + \cos^2(x) = 1$, so $g(x) = 1 + \sin(x)\cos(x)$. If we did not use this then we could differentiate $\sin^2(x)$ and $\cos^2(x)$ using either the product rule or by using the power and chain rules.

To differentiate $g(x)$ we use the product rule on the term $\sin(x)\cos(x)$. So we obtain $g'(x) = 0 + \cos(x)\cos(x) + \sin(x)(-\sin(x)) = \cos^2(x) - \sin^2(x)$. Note that we could also write this as $g'(x) = \cos(2x)$ using the double angle identity for cos.

2. Let $h(x) = \dfrac{\cos(x)}{e^x}$. Find $h'(0)$.

Solution: We find $h'(x)$ using the quotient rule. So $h'(x) = \dfrac{-\sin(x)e^x - \cos(x)e^x}{e^{2x}} = \dfrac{-(\sin(x) + \cos(x))}{e^x}$. So $h'(0) = \dfrac{-(0+1)}{1} = -1$.

3. Find all x-coordinates at which the function $f(x) = 2x^3 + 21x^2 - 48x + 17$ has a horizontal tangent.

Solution: The function $f(x)$ has a horizontal tangent when its derivative equals zero. So we compute $f'(x) = 6x^2 + 42x - 48$ and we set $6x^2 + 42x - 48 = 0$. We can divide this equation by 6 to obtain $x^2 + 7x - 8 = 0$ which we can factor as $(x-1)(x+8) = 0$. So $x = 1$ or $x = -8$. So $f(x)$ has horizontal tangents at $x = 1$ and $x = 8$.