Math 135, Quiz # 4, Febuary 17, 2014

1. Use the definition of the derivative to compute $f'(x)$ if $f(x) = 6x^2 - 5$. Use the derivative to find the slope of the tangent line to the graph of $f(x)$ at the point $(3, 49)$.

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{6(x+h)^2 - 5 - [6x^2 - 5]}{h} \\
&= \lim_{h \to 0} \frac{6x^2 + 12xh + 6h^2 - 5 - 6x^2 + 5}{h} \\
&= \lim_{h \to 0} \frac{12xh + 6h^2}{h} \\
&= \lim_{h \to 0} 12x + 6h \\
&= 12x
\end{aligned}$$

So $f'(x) = 12x$. Therefore the slope at this point is $f'(3) = 12 \cdot 3 = 36$. So using point slope form the equation of the line is $y - 49 = 36(x - 3)$.

2. Find the equation of the secant line for the function $g(x) = 2x^3 + x + 4$ from $x = 1$ to $x = 2$.

Solution: The slope of the secant line is $\frac{g(2) - g(1)}{2 - 1} = \frac{22 - 7}{1} = 15$. So we can use point slope form and the left endpoint $(1, 7)$ to obtain $y - 7 = 15(x - 1)$. Note that we could have used the right endpoint in our equation. Once put in slope-intercept form both will have the same equation.