Math 135, Quiz # 3 Solutions, Febuary 10, 2014

1. Let

$$f(x) = \begin{cases} \frac{x^2-9x+18}{x-3}, & x \neq 3 \\ 2 & x = 3 \end{cases}$$

Is $f(x)$ continuous? Why or why not?

Solution: The function $f(x)$ is *not* continuous. Observe that $f(3) = 2$, however

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2 - 9x + 18}{x - 3} = \lim_{x\to 3} \frac{(x-3)(x-6)}{x-3} = \lim_{x\to 3} x - 6 = 3 - 6 = -3.$$

Since $2 \neq -3$ the limit does not equal the function at $x = 3$ and so the function is discontinous at this point. This type of discountinuity is called a *removable* discontinuity.

2. a) Evaluate $\ln(e^3)$.

Solution: The functions $\ln(x)$ and e^x are inverses, $\ln(e^3) = 3$.

b) Evaluate $e^0 + \ln(1)$.

Solution: First, note that $e^0 = 1$. For any real number α, we define $\alpha^0 = 1$. Next taking the ln of both sides of the equation $e^0 = 1$ we find $0 = \ln(1)$. So $e^0 + \ln(1) = 1 + 0 = 1$.

c) Solve $\log_3(5x - 1) - \log_3(10x + 7) + 1 = 0$.

Solution: If we raise 3 to both sides of the equation we obtain

$$(5x - 1)/(10x + 7) \cdot 3 = 1.$$

Rearranging, $3(5x - 1) = 10x + 7$ which we can simplify as $15x - 3 = 10x + 7 \Rightarrow 5x = 10 \Rightarrow x = 2$. Note, if we plug $x = 2$ back into the equation we find $\log_3(5 \cdot 2 - 1) = \log_3(9) = 2$ and $\log_3(10 \cdot 2 + 7) = \log_3(27) = 3$. So we have $2 - 3 + 1 = 0$ which is true.