Math 135, Quiz # 2 Solutions, Febuary 3, 2014

1. Evaluate $\lim_{x\to 2^+} \dfrac{1}{(x-2)^3}$ and $\lim_{x\to 2^-} \dfrac{1}{(x-2)^3}$. Use these to evaluate $\lim_{x\to 2} \dfrac{1}{(x-2)^3}$.

Solution: As $x \to 2$ from the right $x-2$ becomes a smaller and smaller *positive* number. So $(x-2)^3$ is a smaller and smaller positive number and therefore $1/(x-2)^3$ becomes a large positive number. So $\lim_{x\to 2^+} \dfrac{1}{(x-2)^3} = \infty$.

As $x \to 2$ from the left $x-2$ becomes a smaller and smaller *negative* number. Therefore $1/(x-2)^3$ becomes a larger negative number. So $\lim_{x\to 2^-} \dfrac{1}{(x-2)^3} = -\infty$.

Recall that a limit only exists if the right and left limits match. These limits do not match. So $\lim_{x\to 2} \dfrac{1}{(x-2)^3}$ does not exist.

Notice that the power in the denominator, 3, is an odd number. Thus when we cubed $(x-2)$ when x approached from the left we were cubing a negative number and it stayed negative. What would happen if the power was even? For example, compute: $\lim_{x\to 2} \dfrac{1}{(x-2)^4}$.

2. Evaluate $\lim_{x\to 4} \dfrac{x^2+x-20}{(x-1)^2-9}$.

Solution: If we attempt to plug in $x=4$ to the expression we get $\dfrac{0}{0}$. This suggests we need to do some algebraic simplification and then try evaluating again. We can factor the numerator and denominator as

$$\frac{x^2+x-20}{(x-1)^2-9} = \frac{x^2+x-20}{x^2-2x-8} = \frac{(x-4)(x+5)}{(x-4)(x+2)} = \frac{x+5}{x+2}.$$

Notice it is relatively easy to factor the numerator and denominator since we already know that 4 is a root of each polynomial so we know the factor $(x-4)$ will appear in both factorizations.

So we find $\lim_{x\to 4} \dfrac{x^2+x-20}{(x-1)^2-9} = \lim_{x\to 4} \dfrac{x+5}{x+2} = \dfrac{4+5}{4+2} = \dfrac{9}{6} = \dfrac{3}{2}$.