Math 135, Quiz #12 Solutions, May 5, 2014

1. Let $F(x) = \int_{\pi}^{2x} \cos^2(t)\sin^2(t)\ dt$. Find $F'(x)$.

Solution: By the second fundamental theorem of calculus $F'(x) = \cos^2(2x)\sin^2(2x)\cdot 2$ where the multiplication by 2 comes from the chain rule (the derivative of $2x$ is 2). Note that it does not matter what constant the integral begins at in terms of the deriviative. We could have replaced π by any constant $(0, -4, e)$ and the answer would be the same.

2. Compute the following integrals:

a. $\int (x+1)\sin(x^2+2x-3)\ dx$.

Solution: Let $u = x^2+2x-3$. Then $du = (2x+2)dx$. So $dx = \frac{du}{2x+2}$. Substituting, we have $\int (x+1)\sin(u)\frac{du}{2x+2} = \frac{1}{2}\int \sin(u)du = -\frac{1}{2}\cos(u) + C$. Finally we substitutde back in for x to obtain $-\frac{1}{2}\cos(x^2+2x-3)+C$. Note that if we differentiate this expression we do indeed obtain $(x+1)\sin(x^2+2x-3)$.

b. $\int_0^2 x\sqrt{2x+1}\ dx$.

Solution: Let $u = 2x+1$. Then $du = 2dx$. So $dx = du/2$. When we substitute our bounds change: $0 \to 1$ and $2 \to 5$. The integral becomes $\int_1^5 (u-1)/2\sqrt{u}\ du/2$. Note that we substituted $x = (u-1)/2$. We can simplify this integral as $\frac{1}{4}\int_1^5 \left(u^{3/2} - u^{1/2}\right) du =$ $\frac{1}{4}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_1^5 = \frac{1}{4}\left[(10\sqrt{5} - \frac{10}{3}\sqrt{5}) - \left(\frac{2}{5} - \frac{2}{3}\right)\right] = \frac{5}{3}\sqrt{5} + \frac{1}{15}$.

Note: Apologies that this was a hard problem! What is important (and what I looked for in grading it) was the correct substitution, changing the bounds and breaking up the integral. Beyond that the math gets messy and is challenging to finish correctly on a timed quiz. I didn't take off for algebra/arithmetic mistakes.