Math 135, Quiz #10 Solutions, April 21, 2014

1. Let $C(x) = \frac{1}{8}x^2 + 5x + 98$ be the cost of producing x units of widgets. The manufacturer sets the selling price at $p(x) = \frac{1}{2}(75 - x)$ per widget when x units are produced. What level of widget production optimizes the profit?

Solution:

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We know that profit is maximized when marginal revinue equals marginal cost. The marginal cost is given by $C'(x) = \frac{1}{4}x + 5$. The revinue is $R(x) = x \cdot p(x) = x \cdot \dfrac{1}{2}(75 - x) = \dfrac{1}{2}(75x - x^2)$. Then the marginal revinue is $R'(x) = \dfrac{75}{2} - x$. Equating $C'(x) = R'(x)$ we have $\dfrac{1}{4}x + 5 = \dfrac{75}{2} - x$. Solving, $x = 26$. So producing 26 widgets maximizes profit.

2. a) Find the general antiderivaitve of $f(x) = 4x^3 + x^2 - 3x + e^x$. Don't forget your constant term!

Solution (First read the bolded message accompying problem 1): The general antiderivative is $x^4 + \dfrac{1}{3}x^3 - \dfrac{3}{2}x^2 + e^x + C$ where C is a constant.

b) Suppose $F(x)$ is an antiderivative of $f(x)$ and we know that $F(0) = 5$. What is $F(x)$?

Solution: We know that $F(x) = x^4 + \dfrac{1}{3}x^3 - \dfrac{3}{2}x^2 + e^x + C$ for some constant C. We know that $5 = F(0) = 1 + C$. So $C = 4$. Thus $F(x) = x^4 + \dfrac{1}{3}x^3 - \dfrac{3}{2}x^2 + e^x + 4$.